

# AN ROINN OIDEACHAIS.

(Department of Education).

LEAVING CERTIFICATE EXAMINATION, 1947.

## MATHEMATICS—Algebra—Pass.

MONDAY, 16th JUNE.—MORNING, 10 TO 12.30

Seven questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

1. Solve the following equation for  $x$  and verify your answers :

$$x(x-6) = \frac{648}{x^2-6x} + 63.$$

[28 marks].

2. If

$x^4+6x^3-7x^2-48x-36 \equiv (x^2+ax)^2 - (bx+c)^2$ , where  $a$ ,  $b$  and  $c$  are independent of  $x$ , find the values of  $a$ ,  $b$  and  $c$ . Hence find all the values of  $x$  which satisfy the equation

$$x^4+6x^3-7x^2-48x-36=0.$$

[28 marks.]

3. (i) You are given that

$$f(x) = \frac{1}{2}(e^x + e^{-x}) \text{ and } g(x) = \frac{1}{2}(e^x - e^{-x}).$$

Write down the expressions which are represented by  $f(2x)$  and  $g(2x)$ .

Prove that

$$(a) [f(x)]^2 - [g(x)]^2 = 1; \quad (b) [f(x)]^2 + [g(x)]^2 = f(2x);$$

$$(c) 2f(x) \cdot g(x) = g(2x).$$

$$(ii) \text{ If } \frac{ax}{b-c} = \frac{by}{c-a} = \frac{cz}{a-b}$$

prove that

$$ax+by+cz = a^2x+b^2y+c^2z=0.$$

[28 marks].

4. Express  $(36-12\sqrt{5})^{\frac{1}{2}}$  in its simplest surd form.

Apply a similar method to find in a corresponding surd form the square root of  $(5a+1)+2\sqrt{(6a^2+a-2)}$ , where  $a$  is rational.

[28 marks].

5. Solve the equations

(i)  $xy=6, yz=8, zx=27$ ;

(ii)  $\sqrt{(2x+2)}=1+\sqrt{(x+2)}$ .

[28 marks].

6. Factorize completely

(i)  $(a^2-3a+2)x^2+(2a^2-4a+1)x+a(a-1)$ ;

(ii)  $bc(b+c)+ca(c+a)+ab(a+b)+2abc$ .

[28 marks].

7. On a day on which the wind was blowing at 30 miles per hour directly from a place A to a place B an airplane flew in a straight line from A to B and returned immediately to A, taking 3 hr. 20 min. for the whole journey. On the following day the wind direction was unchanged but its velocity had increased to 50 miles per hour and the double journey then took 3 hr. 36 min. Assuming that the speed of the plane in still air is increased or decreased by the speed of the wind when the plane is flying with or against the wind respectively, find the distance from A to B and the speed of the plane in still air.

[29 marks].

8. (i) Find the arithmetical progression in which the fifteenth term is  $-2\frac{3}{4}$  and the sum of the first fifteen terms is 3.

(ii) Prove that the sum of the odd integers less than 200 is four times the sum of the odd integers less than 100.

[29 marks].

9. (i) In a geometric progression of  $2n$  terms the common ratio is  $r$ . Find the ratio of the sum of the even terms to the sum of the odd terms.

(ii) The sum of the first ten terms of a geometric progression is 2 and the sum of the first twenty terms is 2,050. Find the first term and the common ratio.

[29 marks].

10. Using the same axes and scales draw the graphs of the equations

$$x^2+y^2=25, 2y+x^2=0.$$

Find equations (i) in  $x$ , (ii) in  $y$ , which are satisfied by the co-ordinates of the points of intersection of the two graphs. Find the coordinates of these points of intersection.

[29 marks].