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LEAVING CERTIFICATE EXAMINATION, 1946.

MATHEMATICS.—GEOMETRY.—PASS.

THURSDAY, 13*th* *JUNE*.—AFTERNOON, 3 TO 5.30.

Six questions to be attempted, of which not more than *four* may be selected from Section A.

All questions are of equal value.

Mathematical Tables may be obtained from the Superintendent.

SECTION A.

1. ABC is an acute-angled triangle and AD, BE, CF are the perpendiculars from A, B, C on the sides BC, CA, AB respectively; prove that

(i) AD, BE, CF are concurrent,

(ii) angle EDF$=180°-2$A.

2. Two triangles have the same altitude. Prove that the ratio of their areas is equal to the ratio of their bases.

ABCD is a quadrilateral whose diagonals AC, BD intersect at O. Prove that

$$\triangle ABO : \triangle BCO = \triangle ADO : \triangle DCO.$$

3. Prove that the external bisector of an angle of a triangle divides the opposite side externally in the ratio of the other two sides.

In a triangle ABC, AB$=10$, BC$=4$, CA$=8$; the external bisector of the angle A meets BC produced at X. Calculate the lengths of BX and CX.

4. Show, with proof, how to find a mean proportional between two given straight lines.

Two circles of radius r and R have external contact and one of their common tangents touches the circles at X and Y. Prove that $XY^2=4rR$ and hence show that XY is a mean proportional between their diameters.

5. Show, with proof, how to circumscribe a circle round a given triangle.

Two triangles stand on the same base and on the same side of it and their vertical angles are supplementary. Prove that their circumcircles have equal radii.

6. Given the base BC of a triangle fixed in magnitude and position, and also the magnitude of the vertical angle A. Find the locus of a point P in BA produced which is got by making AP=AC.

Show how to construct a triangle when the following data are given: base, vertical angle and sum of the two sides containing the vertical angle.

Section B.

7. (*a*) If $\sin A=\frac{3}{5}$, $\tan B=\frac{5}{12}$, calculate, without using your tables, the value of each of the following:

(i) $\sin(A+B)$, (ii) $\cos(A+B)$, where A and B are acute angles.

(*b*) Show that

$$\cos\theta+\cos(\theta+120°)+\cos(\theta+240°)=0, \text{ where } \theta \text{ is any angle.}$$

8. In a triangle ABC, $a=17{\cdot}48$, $A=62°\ 12'$, $B=57°\ 9'$; find b and c.

9. Prove that $\sin^2\theta+\cos^2\theta=1$

(i) where θ is an acute angle, (ii) where θ is an obtuse angle.

If

$$x\cos\theta+y\sin\theta+1=0,$$
$$a\cos\theta+b\sin\theta+1=0,$$

solve for $\cos\theta$ and $\sin\theta$ in terms of x, y, a, b.

Hence show that

$$(x-a)^2+(y-b)^2=(bx-ay)^2.$$

10. Draw the graphs of

$$y=\cos^2 x, \text{ and } y=\sin\tfrac{1}{2}x-0{\cdot}4,$$

for values of x from 0° to 180°, using as large a scale as your paper allows.

Hence solve the equation

$$\cos^2 x-\sin\tfrac{1}{2}x+0{\cdot}4=0,$$

for values of x between 0° and 180°.