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(Department of Education).

LEAVING CERTIFICATE EXAMINATION, 1946.

MATHEMATICS—Algebra—Pass.

WEDNESDAY, 12th JUNE—MORNING, 10 TO 12.30.

Seven questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

1. Solve the equation

$$2x^2 - x - 15 = 0.$$

Find *all* the values of y which satisfy the equation

$$2(3y^2 - 8y)^2 - 3y^2 + 8y - 15 = 0.$$

[28 marks.]

2. Express in its simplest surd form $\sqrt{(33 - 12\sqrt{6})}$.

Simplify $[8 + \sqrt{(33 - 12\sqrt{6})}]^{-\frac{1}{2}}$ and find its value correct to three significant figures.

[28 marks.]

3. Find the values of x and y which satisfy the simultaneous equations

$$x^2 + y^2 - 10x + 4y + 4 = 0.$$

$$x - y - 8 = 0.$$

[28 marks.]

4. (i) Find all the factors for

$$x^4 + 2x^3 - 3x^2 + 2x + 6.$$

(ii) If

$$4x^4 + 4x^3 - 3x^2 + ax + b = (px^2 + qx + r)^2,$$

where a, b, p, q, r are independent of x , determine the values of the coefficients and factorise the expression on the left-hand side completely.

[28 marks.]

5. If

$$\begin{aligned} 3x/a + y/b - 4z/c &= 0 \\ 5bcx - 7cay + 2abz &= 0. \end{aligned}$$

prove that

$$x/a = y/b = z/c$$

and, as a deduction from that, show that

$$(a + b + c)(ax + by + cz) = (x + y + z)(a^2 + b^2 + c^2).$$

[28 marks.]

6. ABC is a right-angled triangle, the angle at C being a right angle. The perimeter of the triangle is 112 in. and the hypotenuse AB is 2 in. longer than the side AC. Find the sides of the triangle and the length of the perpendicular from C to AB.

[28 marks.]

7. (i) Explain clearly why we write

$$a^0=1 \text{ and } a^{-3}=1/a^3.$$

(ii) Find the value of x in each of the following cases :

(a) $\log_2(\frac{1}{4})=x$; (b) $\log_4x=\frac{1}{2}$; (c) $\log_44=-\frac{1}{2}$; (d) $2^{x+1}=5$.

[29 marks.]

8. (i) What is the n th term in each of these series :

(a) $1+3+5+7+\dots$,

(b) $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\dots$,

(c) $1 \times 3+3 \times 5+5 \times 7+7 \times 9+\dots?$

(ii) The first term in an arithmetical progression is -7 and the sum of the first 22 terms is zero. Find the series. How many terms of the series must be taken so that their sum is 16?

[29 marks.]

9. (i) Find, from first principles, the sum of n terms of the series

$$a^2+a^2r^2+a^2r^4+a^2r^6+\dots$$

(ii) What is the least number of terms of the series

$$11+3\frac{2}{3}+1\frac{2}{9}+\dots$$

which must be taken so that their sum may be greater than 16.499?

[29 marks.]

10. Using the same axes and scales draw graphs, between $x=-2$ and $x=+4$, of the functions

$$2(x+1)(x-3) \text{ and } x(x+1)(x-3).$$

For what values of x are the two functions equal? For what values of x in the given range does the second function exceed the first algebraically?

[29 marks.]