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LEAVING CERTIFICATE EXAMINATION, 1945.

MATHEMATICS.—GEOMETRY.—PASS.

WEDNESDAY, 13*th* *JUNE*.—AFTERNOON, 3 TO 5.30.

Six questions to be attempted, of which not more than *four* may be selected from Section A.

All questions are of equal value.

Mathematical Tables may be obtained from the Superintendent.

SECTION A.

1. From a point P, outside a circle, a tangent PT and a secant PBA are drawn. Prove that the angle PTB is equal to the angle PAT.
A straight line is drawn parallel to PT meeting TA and TB at X, Y respectively; prove that $XYBA$ is a cyclic quadrilateral.

2. Prove that equiangular triangles are similar.
ABC is a triangle inscribed in a circle. The bisector of the angle A meets the base in D and meets the circumference of the circle in E. Show that the triangles ADB, ACE are similar and hence prove that
$$AB.AC = BD.DC + AD^2.$$

3. ABC, DEF are two similar triangles, having BC, EF as corresponding sides. Prove that
$$\triangle ABC : \triangle DEF :: BC^2 : EF^2.$$
Deduce the corresponding theorem in the case of two similar quadrilaterals.

4. If a straight line is drawn parallel to one side of a triangle, prove that it divides the other two sides proportionally.
Prove that three parallel straight lines cut any two intersecting straight lines proportionally.

5. Show how to construct a circle touching one side of a triangle and touching the other two sides produced.
If the triangle is equilateral and the length of each side is a, prove that $4r^2 = 3a^2$, where r is the radius of the above circle.

6. ABC is a triangle having the base BC fixed in magnitude and position. If the angle A is fixed in magnitude, what is the locus of the point A?
If the bisector of the angle A meets the circumference of the circle circumscribed about ABC at the point X, show that $IX = XC$, where I is the centre of the circle inscribed in ABC. Hence deduce the locus of I as A varies in position.

Section B.

7. From the top of a tower, 155 ft. high, the angles of depression of two points which are in the same vertical plane as the observer are $28^\circ\ 16'$ and $44^\circ\ 19'$. If the two points are in the horizontal plane passing through the foot of the tower, calculate their distance apart.

8. Using the formulae for $\sin(A+B)$ and $\cos(A+B)$, show that

(i) $\sin 2A = 2\sin A\cos A$,

(ii) $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1$.

Deduce that

$\sin 3A = 3\sin A - 4\sin^3 A$,
$\cos 3A = 4\cos^3 A - 3\cos A$.

9. With the usual notation, show that the area of a triangle ABC is equal to $\frac{1}{2}ab\sin C$.

A regular pentagon is inscribed in a circle of radius 5 cm. Using the Tables, calculate (i) the area, (ii) the perimeter, of the pentagon.

10. Make out a table of values of $3\sin x + 4\cos x$ when x has the values

0°, 30°, 45°, 60°, 90°, 120°, 135°, 150°, 180°.

Hence draw the graph of

$$y = 3\sin x + 4\cos x.$$

From your graph,

(i) find the maximum value of $3\sin x + 4\cos x$,

(ii) solve the equation $3\sin x + 4\cos x = 2\frac{1}{2}$.