AN ROINN OIDEACHAIS
(Department of Education).

BRAINNSE AN MHEADHON-OIDEACHAIS
(Secondary Education Branch).

LEAVING CERTIFICATE EXAMINATION, 1937.

PASS.

MATHEMATICS
(GEOMETRY)

THURSDAY, 17*th* *JUNE*.—MORNING, 10 A.M. TO 12.30 P.M.

Six questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

Candidates should state the text-book used in order to indicate the sequence followed.

1. Prove that the medians of a triangle are concurrent and that they divide one another in the ratio 2 : 1.

D, E, F are the mid-points of the sides BC, CA, AB respectively of a triangle ABC: construct the triangle so that AB=3 ins., AD=1·8 ins., CF=2·7 ins. [30 marks.]

2. Prove that if triangles are equiangular their corresponding sides are proportional.

Two circles A, B of diameters a, b touch externally at P. Through P a line is drawn intersecting A, B at Q, R respectively: prove that PQ : PR=a : b. [30 marks.]

3. Prove that the area of a triangle is equal to $\frac{1}{2}bc\sin A$, where b, c are the lengths of two of the sides and A is the contained angle.

ABC is a triangle right-angled at A. A parallelogram PQRS is equal in area to ABC and its sides PQ, PS are equal to AB, AC respectively: find the number of degrees in each of its angles. [30 marks.]

4. ABC is a triangle: prove the equation connecting BC^2 with AB^2, AC^2 and cosA when A (i) is an acute angle, (ii) is an obtuse angle.

Find the number of degrees in the smallest angle of a triangle whose sides are 1·6 ins., 2·2 ins., 1·8 ins in length.

[30 marks.]

5. A line AB, one foot in length, is divided internally at P and externally at P_1 so that $AB.AP=BP^2$ and $AB.AP_1=BP_1{}^2$. Express the lengths of AP, AP_1 in simplest surd form.

Give geometrical constructions for finding the points P and P_1. (Proof need not be given but the construction should be clear.)

[30 marks.]

6. Prove that if a polygon inscribed in a circle be *equilateral* it must also be *equiangular* but that the converse theorem is not always true.

[30 marks.]

7. Prove that the bisector of an angle of a triangle divides the opposite side into segments which are proportional to the other two sides of the triangle.

Show how to construct a triangle being given the base, the vertical angle and the point at which the bisector of the vertical angle meets the base.

[35 marks.]

8. Prove that the rectangle contained by the diagonals of a cyclic quadrilateral is equal to the sum of the rectangles contained by the opposite sides.

O is the orthocentre of a triangle ABC, AD is the perpendicular from A on BC, while L, M, N are the mid-points of OA, OB, OC respectively: prove that 2BC.LD=AB.CO+AC.BO.
(*Hint:* Use the cyclic quadrilateral LMDN).

[35 marks.]

9. With the usual notation for a triangle ABC prove that $a=2R\sin A$ where R is the circumcircle of the triangle.

Two angles of a triangle contain 45° and $22\frac{1}{2}$° respectively and the radius of the circumcircle is 3 ins. Calculate the length of the longest side.

[35 marks.]

10. ABC is any triangle: prove that $\tan\frac{1}{2}(B-C)=\frac{b-c}{b+c}\cot\frac{A}{2}$.

If $b=387$, $c=259$, $A=98°$, calculate the values of B and a.

[35 marks.]