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LEAVING CERTIFICATE EXAMINATION, 1935.

PASS.

MATHEMATICS
(ALGEBRA)

TUESDAY, 18*th* *JUNE*.—AFTERNOON, 3.30 TO 6 P.M.

Seven questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

1. Prove that $x-2$ is a factor of $3x^3+x^2-34x+40$. Find the other factors and hence solve the equations

   (i) $3x^3+x^2-34x+40=0$

   (ii) $3(x-4)^3+(x-4)^2-34(x-4)+40=0$.

[28 marks.]

2. Find a number of three digits such that the product of the hundreds' digit and the units' digit is four times the tens' digit and such that the number obtained by interchanging the hundreds' digit and the units' digit exceeds the original number by 495, while the number obtained by interchanging the hundreds' digit and the tens' digit exceeds the original number by 270.

[28 marks.]

3. Solve the equations

   (i) $\left.\begin{array}{l} x^2+xy+y^2=19 \\ x^2-xy+y^2=\ 7 \end{array}\right\}$

   (ii) $3\sqrt{x+9}-2\sqrt{2x-10}=\sqrt{9x+1}$.

[28 marks.]

4. Three motor-cars, A, B, C, left Dublin at 1 p.m., 1.30 p.m. and 2 p.m. respectively, travelling in the same direction along the same road. B's speed exceeded A's by 5 miles per hour and C's speed exceeded B's by 6 miles per hour. When B overtook A, C was three miles behind both: find when C overtook A and B.

[28 marks.]

5. Solve the equation $lx^2+mx+n=0$. Deduce that

$$\alpha+\beta=-\frac{m}{l},\ \alpha\beta=\frac{n}{l},$$

where α, β are the roots.

Form the equation whose roots are

$$\frac{\alpha+1}{\alpha-1} \text{ and } \frac{\beta+1}{\beta-1}.$$

[28 marks.]

6. Express the square root of $2+\sqrt{3}$ in the form $\sqrt{a}+\sqrt{b}$, where a and b are rational numbers.

[28 marks.]

7. How many of the numbers

1, 2, 3, 4, 5, 6, 7 . . . . . . 998, 999, 1000

are *not* divisible by 7 ?

Find the sum of all the positive integers less than 1000 which are *not* divisible by 7.

[29 marks.]

8. AOB ($=\theta$) is an acute angle and P is a point on OA such that OP$=x$ ins. From P a perpendicular PP_1 is drawn to OB ; from P_1 a perpendicular P_1P_2 is drawn to OA; from P_2 a perpendicular P_2P_3 is drawn to OB ; and so on. Prove that PP_1 P_1P_2, P_2P_3, P_3P_4, etc., are in Geometrical Progression and calculate the length of P_2P_3 when $\theta=25°$, $x=10$.

[29 marks.]

9. Using squared-paper and the same axes and scales draw the graphs of

$$y=3x^2-5x-4 \text{ and } y=7-2x.$$

From your graphs, or otherwise, determine approximately for what range of values of x the expression $3x^2-5x-4$ is less than $7-2x$.

[29 marks.]

10. Prove that

$$a^3+b^3+c^3-3abc=\tfrac{1}{2}(a+b+c)\left[(b-c)^2+(c-a)^2+(a-b)^2\right].$$

Hence show that

$$a^3+b^3+c^3-3abc=(x^3+y^3+z^3-3xyz)^2$$
$$\text{when } \left.\begin{aligned} a&=x^2-yz \\ b&=y^2-zx \\ c&=z^2-xy \end{aligned}\right\}$$

[29 marks.]