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LEAVING CERTIFICATE EXAMINATION, 1930.

PASS.

MATHEMATICS (I).

FRIDAY, 13th JUNE.—MORNING, 10 A.M. TO 12.30 P.M.

Seven questions may be answered. 8 (*a*) or 8 (*b*) may be answered, but not both. All questions carry equal marks.

Mathematical Tables may be obtained from the Superintendent.

1. Solve the equations :

$$(a)\quad \frac{2}{1+x}+\frac{3}{1-x}=5.$$

$$(b)\quad \left.\begin{aligned}\frac{1}{x}+\frac{1}{y}&=a\\ xy&=b^2\end{aligned}\right\}$$

Test your solutions in (*a*).

2. Find, in its simplest form, correct to *five* significant figures, the value of

$$\sqrt{\left(\frac{1}{7-4\sqrt{3}}\right)}\quad -\sqrt{\left(\frac{1}{5+2\sqrt{6}}\right)}.$$

3. State and prove the *Remainder Theorem.*

Show that $x^n-nx+n-1$ is divisible by $(x-1)^2$, where n is any positive integer greater than unity.

4. Define a *logarithm,* and from the definition establish that $\log_a\frac{M}{N}=\log_a M-\log_a N$.

If $\log_a(p+q-r)=\log_a p+\log_a q-\log_a r$, find the simplest relations between p, q, r.

5. Arithmetic Means, whose sum is 132, are inserted between 1 and 21 : find the first two of those means.

6. A rectangle, of area one square foot, has diagonal, length and breadth in Geometric Progression : find the angle between the diagonal and the longer side.

7. A man paying a coal bill observed that $1\frac{1}{2}$ tons less coal would have been obtained for the money had the price been $12\frac{1}{2}\%$ higher, but that $11\frac{1}{9}\%$ more coal would have been obtained if the price had been 4*s*. per ton lower : what was the amount of the bill ?

8 (*a*) Prove the identity

$$(a^2+b^2+c^2)(x^2+y^2+z^2)=(ax+by+cz)^2+(bz-cy)^2+(cx-az)^2+(ay-bx)^2.$$

Hence show that, if $a^2+b^2+c^2$ and $x^2+y^2+z^2$ have given values, $ax+by+cz$ will have its maximum value when $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}$.

Or

8 (*b*) Show that the coefficient of the middle term of the expansion of $(1+x)^{16}$ is equal to the sum of the coefficients of the eighth and ninth terms of the expansion of $(1+x)^{15}$.

Use the Binomial Theorem to evaluate $\sqrt[5]{40}$ to *three* places of decimals.

9. ABC is a triangle right-angled at C. Through any point P on AB lines PD and PE are drawn parallel to CA and BC respectively, and forming with them the rectangle PDCE. If BC is *a* units in length, CA *b* units and PE *x* units, express the length of PD in terms of *a*, *b*, *x*, calculate the area of PDCE and show that it cannot be greater than $\frac{1}{4}ab$.

10. Plot the graphs of $x=\frac{6}{y}-y$ and of $y=3x-\frac{1}{x}$ for values of x lying between -3 and 3, and thus obtain solutions of the equations

$$\left.\begin{aligned} xy+y^2 &= 6 \\ 3x^2-xy &= 1 \end{aligned}\right\}$$