AN ROINN OIDEACHAIS

(Department of Education).

BRAINSE AN MHEÁN-OIDEACHAIS

(Secondary Education Branch).

LEAVING CERTIFICATE EXAMINATION, 1928.

PASS

MATHEMATICS (II).

MONDAY, 18*th* *JUNE*.—Afternoon, 3.30 to 6 p.m.

Six questions may be answered. All questions carry equal marks.

Mathematical Tables may be obtained from the Superintendent.

Candidates should state the text-book used in order to indicate the sequence followed.

1. Show that the perpendicular bisector of a chord of a circle passes through the centre.

If two chords AB and CD cut at right-angles in the point O, show that the sum of the squares on OA, OB, OC and OD is equal to the square on the diameter of the circle.

2. Show that in a triangle in which A is acute

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

ABDX, ACFY are squares (named in order) on the sides of triangle ABC. Show that $XY^2 + BC^2 = 2\ (AB^2 + AC^2)$ and that XY is double the median from A to the mid-point of BC.

3. Show how to describe on a given line a segment of a circle to contain a given angle. Give proof.

On a straight line of length $2\frac{1}{2}$ inches construct a triangle of vertical angle 45°, the sum of the other sides being 5 inches.

4. Define a regular polygon and prove that a circle can be inscribed in any regular polygon.

Find how many diagonals of different lengths there are in a regular polygon of n sides.

5. Show that the internal and the external bisectors of the vertical angle of a triangle divide the base in the same ratio.

Hence express the distance between the points of section of the base in terms of a, b, c the sides of the triangle.

6. Show that the rectangle contained by the diameter of the circumscribing circle of a triangle and the perpendicular from the vertex to the base is equal to the rectangle contained by the other two sides.

In a right-angled triangle show that the sum of the hypotenuse and the perpendicular to it from the opposite vertex is greater than the sum of the other two sides. (Use the squares of the sums).

7. Prove that

(i) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$, A and B being acute.

(ii) $\tan(A-B) + \tan(B-C) + \tan(C-A) =$
$\tan(A-B)\tan(B-C)\tan(C-A)$.

8. In each case find a relation between A and B so that

(i) $\cos^2 A + \cos^2 B = 2(1 - \sin A \sin B)$.

(ii) $2\cos^2 A = 1 + \sin 2B$.

Investigate for a triangle the truth of the relation :—

$$2c\cos\tfrac{1}{2}(A-B) = (a+b)\sin\tfrac{1}{2}C.$$

9. From the ends P, Q of a base line 367 yards long the bearings of the foot R of a tower are such that $\angle RPQ = 49°17'$ and $\angle RQP = 53°41'$: the elevation of the tower from P is $10°12'$. Calculate the height of the tower.