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(Department of Education).

BRAINSE AN MHEÁN-OIDEACHAIS

(Secondary Education Branch).

LEAVING CERTIFICATE EXAMINATION, 1927.

PASS

MATHEMATICS (II).

TUESDAY, 21*st* *JUNE*.—AFTERNOON, 3.30 TO 6 P.M.

Six questions may be answered. All questions carry equal marks.

Trigonometrical (Logarithmic) Tables may be obtained from the Superintendent.

1. Show geometrically how to find a point (i) in a line, (ii) in a line produced, so that the rectangle contained by the segments may have a given area.

Between what limits (if any) must the given area lie?

2. AD is a median of the triangle ABC: prove that

$$AB^2 + AC^2 = 2\ (BD^2 + AD^2).$$

Hence find what is the locus of a point which moves so that the sum of the squares on the lines joining it to two fixed points exceeds twice the square on the line joining it to another fixed point by a given amount.

3. Show that a quadrilateral is cyclic if the opposite angles are supplementary.

AB is a diameter and PQ a chord of a circle: AP and BQ meet in M and AQ and BP in N. Show that a circle can be circumscribed about PNQM, and that it touches the radii of the first circle passing through P and Q.

4. Describe, with ruler and compass only, a regular pentagon in a circle of radius 2 inches. (All construction lines should be shown).

5. Prove that a line parallel to one side of a triangle divides the other sides in equal ratios.

AB, of length a, and CD, of length b, are parallel sides of a trapezium ABCD, the points being taken in order. E is the intersection of the diagonals and FEG, a parallel to AB, meets AD in F and BC in G: prove that:

$$\mathrm{FG} = \frac{2ab}{a+b}.$$

6. Prove that

$$\sin(\mathrm{A}-\mathrm{B}) = \sin \mathrm{A} \cos \mathrm{B} - \cos \mathrm{A} \sin \mathrm{B}$$

when A is an acute angle greater than B.

If $$\frac{\sin\theta}{\sin(\alpha+\theta)} = \frac{\sin(\alpha-\theta)}{\sin\theta}$$

prove that $\cos 2\theta = \cos^2\alpha$.

7. In triangle ABC AB is less than AC. A circle of centre A and radius AB cuts CA in D, CA produced in D′. Express angles ADB and DBC in terms of the angles of triangle ABC and prove from the figure, or otherwise, that

$$\tan \tfrac{1}{2}(\mathrm{B}-\mathrm{C}) = \frac{b-c}{b+c} \cot \frac{\mathrm{A}}{2}.$$

8. Solve fully the triangle ABC in which

$$b = 215{\cdot}8,\ c = 124{\cdot}8,\ \mathrm{A} = 47^\circ\ 15'.$$

9. Prove that in any triangle

$$\frac{a+c}{b} = \frac{\cos\frac{1}{2}(\mathrm{A}-\mathrm{C})}{\sin\frac{1}{2}\mathrm{B}}.$$

Hence show that, if C is a right-angle,

$$\tan\tfrac{1}{2}\mathrm{A} = \frac{a-b+c}{a+b+c}.$$