AN ROINN OIDEACHAIS

(Department of Education).

BRAINSE AN MHEÁN-OIDEACHAIS

(Secondary Education Branch).

LEAVING CERTIFICATE EXAMINATION, 1926.

PASS

MATHEMATICS (II).

MONDAY 21st JUNE.—AFTERNOON 3.30 TO 6 P.M.

Six questions may be answered. All questions carry equal marks.

Trigonometrical (Logarithmic) Tables may be obtained from the Superintendent.

1. In what ratio is the base of a triangle divided by the bisector of the opposite angle? Give proof.

Show that the intersection of this bisector with the perpendicular bisector of the base cannot lie within the triangle.

2. Show how to construct an isosceles triangle having each of the base angles double of the vertical angle.

ABC is such a triangle having AB = AC. D is a point in AB such that AD = BC. Show that the triangles ACD and ABC have equal circumscribed circles.

3. Show that the angles contained by a tangent and a secant drawn through a point on a circle are equal to the angles in the alternate segments of the circle.

The inscribed circle of $\triangle\ A_1B_1C_1$ touches the sides in A_2, B_2, C_2. Express the angles of triangle $A_2B_2C_2$ in terms of those of $A_1B_1C_1$.

4. Find the locus of the mid-points of equal chords of a circle.

Show, stating all steps of the construction, how to draw a line intersecting two unequal circles so that the intercepted chords may be of given lengths.

5. Describe an equilateral triangle equal in area to a square of side 2 inches. Prove your construction.

6. Show how to describe an equilateral triangle, being given the difference between the side and the perpendicular height. (The use of the protractor is not permitted.)

If d be the difference, show that the area of the equilateral triangle is $\sqrt{3}.\, d^2 \tan^2 75°$.

7. Find the locus of a point which moves so that its distances from two fixed points are in a constant ratio.

Given the base of a triangle and the ratio of the other two sides, show how to construct the triangle of greatest area.

8. Prove that

$$\text{Sec}^6\,\theta - \tan^6\,\theta = 1 + 3\tan^2\theta\,\text{Sec}^2\,\theta$$

and find, in its simplest form, x from the equation

$$x^2 + (\tan\alpha + \cot\alpha)\,x + 1 = 0.$$

9. Prove that

$$\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

and find the greatest angle in the triangle having sides 7, $8\frac{1}{2}$, 9.