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LEAVING CERTIFICATE EXAMINATION, 1926.

PASS
MATHEMATICS (I).

THURSDAY, 17*th* *JUNE*.—MORNING 10 A.M. TO 12.30 P.M.

Seven questions may be answered. 9 (*a*) *or* 9 (*b*) may be attempted, but not both. All questions carry equal marks.

Tables of Measures, Constants and Formulae, and Logarithm Tables may be obtained from the Superintendent.

1. Solve the equations :

(*a*) $x^2 - 2x(a^2 + b^2) + (a^2 - b^2)^2 = 0$

(*b*) $x + 2y = 5 = \frac{1}{x} + \frac{2}{y}$.

2. Given $s = \dfrac{9\,k\,m}{9k - 2\,(k-1)^2\left(\frac{b^3}{a^3} - 1\right)}$,

Find k, when $s = 11 \cdot 34$, $m = 8 \cdot 4$, $b = 2a$.

3. The sum of the ages of the boys of a class is 247 years. Four new pupils of average age $8\frac{1}{4}$ years join the class, thus reducing the average age of the whole class by two months. Find the original number of pupils in the class.

4. Given the terms of an arithmetical progression, state in words what steps you would follow to find the sum of the series.

The sum of n terms of a series is $3n^2 + 2n$; find the nth term and the first four terms.

5. Factorise

(*a*) $2p^2 - 4pq + 2q^2 - 3p + 3q + 1$.

(*b*) $(x + a)^3 + (x + b)^3 - (2x + a + b)^3$.

Prove that

$$\left(\frac{x}{y} + \frac{y}{z} + \frac{z}{x}\right)\left(\frac{x}{z} + \frac{y}{x} + \frac{z}{y}\right) - \left(\frac{x}{y} + \frac{y}{z}\right)\left(\frac{y}{z} + \frac{z}{x}\right)\left(\frac{z}{x} + \frac{x}{y}\right) \equiv 1.$$

6. Prove that $$\log_b N = \frac{\log_a N}{\log_a b}$$

Find $\log_3 623$, and find the index of the power of 7 which does not differ from 1000 by more than one unit.

7. If $x = \frac{\sqrt{2}-1}{\sqrt{2}+1}$ show that $x\ (6-x) = 1$ and hence or otherwise find the simplest numerical value of

$$3x^3 - 14x^2 - 21x + 29.$$

8. ABC is a triangle right angled at C. The length of the perpendicular from C on AB is p, and AC $= b$ and BC $= a$.

Prove that $$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

9. (*a*) Write down the first three and the last three terms in the expansion of $(x+y)^{13}$.

What are the first 4 terms in the expansion of $(1+2x)^{13}$?

Find to 4 places of decimals the value of $(\cdot 99)^{10}$ without using tables ;

or

(*b*) Write $ax^2 + bx + c$ in the form $a\ (x+d)^2 + f$ where a, b, c, d, f are quantities independent of x.

What sign should a have in order that the expression should have (i) a finite maximum (ii) a finite minimum value ? Give the value in each case.