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LEAVING CERTIFICATE EXAMINATION, 1925

PASS.

MATHEMATICS (I).

WEDNESDAY, 17*th* *JUNE*.—Morning, 10 a.m. *to* 1 p.m.

[Eight questions only to be answered.]

[Note.—*Tables of Measures, Constants and Formulae, and Logarithm Tables may be obtained from the Superintendent.*]

1. If $k = \dfrac{64\pi^2 a^2 l\,(W - w)}{g\,(T_1^2 - T_2^2)}$ and $n = \dfrac{2k}{\pi r^4}$,

find n when $a = 5{\cdot}62$, $l = 94{\cdot}8$, $W = 1346$, $w = 36$, $g = 981$, $T_1 = {\cdot}843$, $T_2 = {\cdot}351$, $r = {\cdot}0487$.

2. Solve the equations

$$x + 2y + 3z = 4,$$
$$4x - 2y + 2z = 1,$$
$$x^2 + 2y^2 + 3z^2 - 6x + 4y = 173,$$

giving x, y, z to two places of decimals.

3. Draw a line AB, a inches long, and on it describe a semicircle. At B erect a perpendicular $BP = \sqrt{b}$ inches. Draw PR_1R_2 parallel to AB, cutting the semicircle at R_1R_2. Drop perpendiculars R_1P_1 and R_2P_2 on AB. Show that the roots of $x(a - x) = b$ are represented by AP_1 and AP_2.

Find geometrically the roots of $x(9 - x) = 4{\cdot}5$.

4. If N is any positive number and n a power of 2, what value or values will N^n and $\sqrt[n]{N}$ approach as n increases indefinitely

(1) when N is greater than 1;

(2) when N is less than 1;

(3) when $N = 1$?

Sum to infinity (without using a formula)

$$9 + 2\tfrac{1}{4} + \tfrac{9}{16} + \text{etc.}$$

5. Write down the expansions of $(a + b)^4$ and $(x + y)^7$.

If, in question 1, an error of ·1 per cent. is made in the measurement of r, and k is known accurately, what would be the percentage error in the calculated value of n?

6. Factorize fully the following:—

(a) $(x + 1)(x + 2)(x + 3)(x + 4) - 360$.

(b) $(ax - by - cz)^3 + (ay - bz - cx)^3 + (az - bx - cy)^3$

if $a = b + c$.

7. Show that $x + \dfrac{a^2}{x} = 2a + \left(\sqrt{x} - \dfrac{a}{\sqrt{x}}\right)^2$.

(a) If x is a positive quantity, what is the least value of $x + \dfrac{a^2}{x}$? What is then the value of x?

(b) If the product of two whole numbers is 324, what is the least and what the greatest sum they can have?

8. AC is a straight railway line and B is a place 5 miles distant from the nearest point C on it. Suppose goods to be sent from A to a point S by rail and thence by a straight road to B, S being x miles from C. The cost per mile by rail being 1·5 shillings and by road 2·5 shillings, and the distance AC 12 miles, give an expression for the total cost of carriage.

Draw a graph showing the cost of carriage for different values of x, and find where S should be so that the cost should be as small as possible.

9. H is the point of contact of a tangent AH to a circle with centre C and radius r. The shortest distance from A to the circle is h, and $AH = d$.

Prove that $d^2 = h^2 + 2hr$.

Find d if A is an aeroplane 1 mile high above the earth and $r = 3960$ miles, the radius of the earth.

Show that if h^2 be neglected, the square of the number of miles in d is approximately equal to $1\frac{1}{2}$ times the number of feet in h.

Use this rule to find the distance AH of the horizon when the aeroplane is at a height of 8000 feet.