Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2015

Mathematics

Paper 1

Higher Level

Friday 5 June Afternoon 2:00 – 4:30

300 marks

Examination number

Centre stamp

Running total	

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. You may ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if all necessary work is not clearly shown.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in simplest form, where relevant.

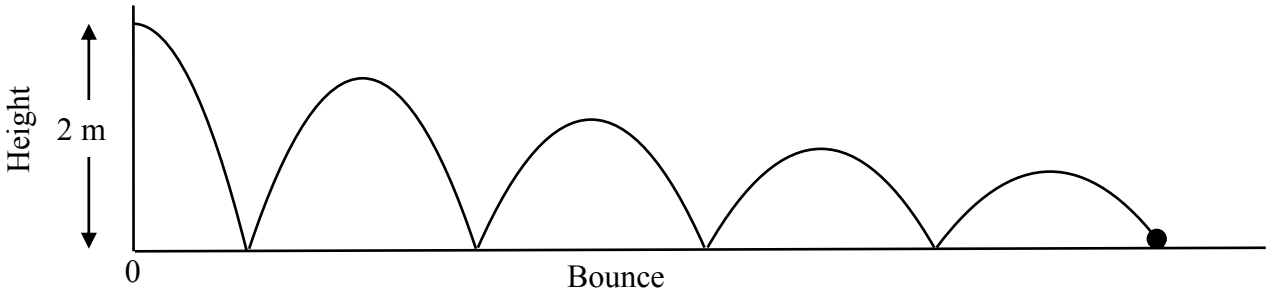
Write the make and model of your calculator(s) here:

Section A — Concepts and Skills — 150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

Mary threw a ball onto level ground from a height of 2 m. Each time the ball hit the ground it bounced back up to $\frac{3}{4}$ of the height of the previous bounce, as shown.



(a) Complete the table below to show the maximum height, **in fraction form**, reached by the ball on each of the first four bounces.

Bounce	0	1	2	3	4
Height (m)	$\frac{2}{1}$				

(b) Find, in metres, the total vertical distance (up and down) the ball had travelled when it hit the ground for the 5^{th} time. Give your answer in fraction form.

(c) If the ball were to continue to bounce indefinitely, find, in metres, the total vertical distance it would travel.

Question 2 **(25 marks)**

Solve the equation $x^3 - 3x^2 - 9x + 11 = 0$.

Write any irrational solution in the form $a + b\sqrt{c}$, where $a, b, c \in \mathbb{Z}$.

Question 3 **(25 marks)**

Let $f(x) = -x^2 + 12x - 27,\ x \in \mathbb{R}$.

(a) **(i)** Complete Table 1 below.

Table 1							
x	3	4	5	6	7	8	9
$f(x)$	0	5			8		

(ii) Use Table 1 and the trapezoidal rule to find the approximate area of the region bounded by the graph of f and the x-axis.

(b) **(i)** Find $\int_3^9 f(x)dx$.

(ii) Use your answers above to find the percentage error in your approximation of the area, correct to one decimal place.

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Question 4 **(25 marks)**

(a) The complex numbers z_1, z_2 and z_3 are such that $\frac{2}{z_1} = \frac{1}{z_2} + \frac{1}{z_3}$, $z_2 = 2 + 3i$ and $z_3 = 3 - 2i$, where $i^2 = -1$. Write z_1 in the form $a + bi$, where $a, b \in \mathbb{Z}$.

(b) Let ω be a complex number such that $\omega^n = 1$, $\omega \neq 1$, and $S = 1 + \omega + \omega^2 + \cdots + \omega^{n-1}$. Use the formula for the sum of a finite geometric series to write the value of S in its simplest form.

Question 5 **(25 marks)**

(a) Solve the equation $x = \sqrt{x+6},\ x \in \mathbb{R}$.

(b) Differentiate $x - \sqrt{x+6}$ with respect to x.

(c) Find the co-ordinates of the turning point of the function $y = x - \sqrt{x+6},\ x \geq -6$.

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Question 6 **(25 marks)**

(a) Donagh is arranging a loan and is examining two different repayment options.

(i) Bank A will charge him a monthly interest rate of 0·35%. Find, correct to three significant figures, the annual percentage rate (APR) that is equivalent to a monthly interest rate of 0·35%.

(ii) Bank B will charge him a rate that is equivalent to an APR of 4·5%. Find, correct to three significant figures, the monthly interest rate that is equivalent to an APR of 4·5%.

(b) Donagh borrowed €80 000 at a monthly interest rate of 0·35%, fixed for the term of the loan, from Bank A. The loan is to be repaid in equal monthly repayments over ten years. The first repayment is due one month after the loan is issued. Calculate, correct to the nearest euro, the amount of each monthly repayment.

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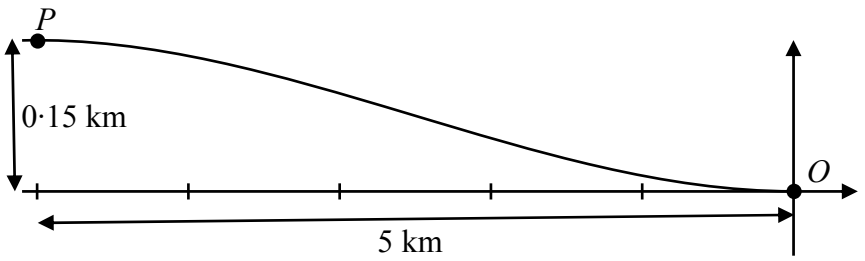
Section B	Contexts and Applications	150 marks

Answer **all three** questions from this section.

Question 7 **(50 marks)**

A plane is flying horizontally at P at a height of 150 m above level ground when it begins its descent. P is 5 km, horizontally, from the point of touchdown O. The plane lands horizontally at O.



Taking O as the origin, $(x,\ f(x))$ approximately describes the path of the plane's descent where $f(x) = 0{\cdot}0024x^3 + 0{\cdot}018x^2 + cx + d,\ -5 \le x \le 0,$ and both x and $f(x)$ are measured in km.

(a) **(i)** Show that $d = 0$.

(ii) Using the fact that P is the point $(-5,\ 0{\cdot}15),$ or otherwise, show that $c = 0$.

(b) **(i)** Find the value of $f'(x),$ the derivative of $f(x),$ when $x = -4$.

(ii) Use your answer to part **(b) (i)** above to find the angle at which the plane is descending when it is 4 km from touchdown. Give your answer correct to the nearest degree.

(c) Show that $(-2{\cdot}5,\ 0{\cdot}075)$ is the point of inflection of the curve $y = f(x)$.

(d) **(i)** If (x, y) is a point on the curve $y = f(x)$, verify that $(-x-5, -y+0{\cdot}15)$ is also a point on $y = f(x)$.

(ii) Find the image of $(-x-5, -y+0{\cdot}15)$ under symmetry in the point of inflection.

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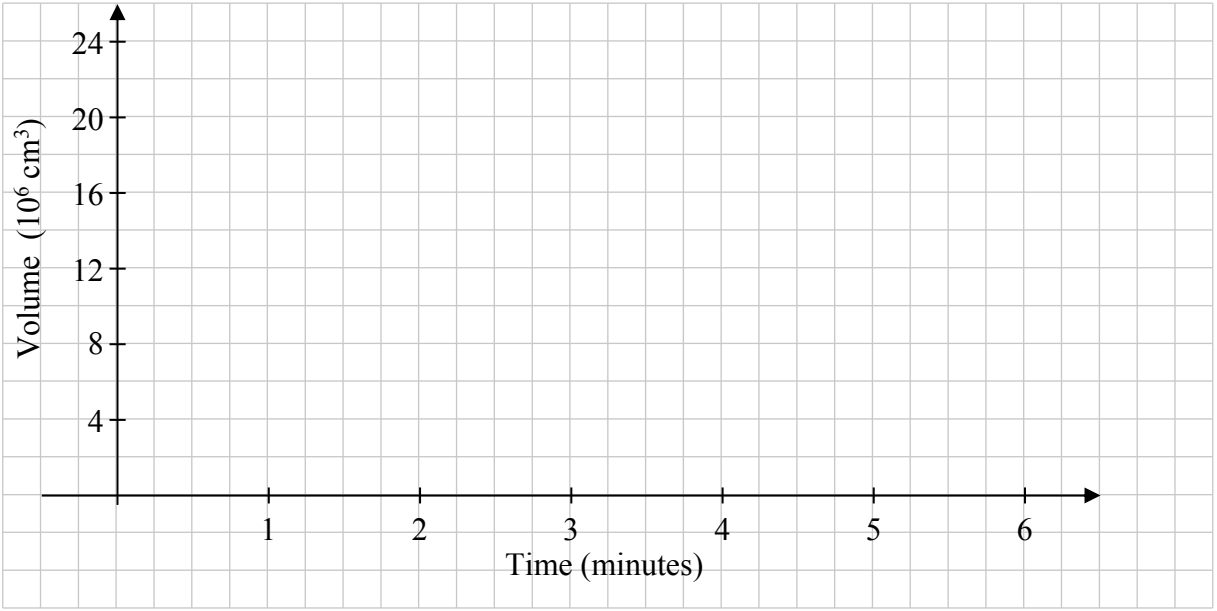
Question 8 **(50 marks)**

An oil-spill occurs off-shore in an area of calm water with no currents. The oil is spilling at a rate of 4×10^6 cm^3 per minute. The oil floats on top of the water.

(a) **(i)** Complete the table below to show the total volume of oil on the water after each of the first 6 minutes of the oil-spill.

Time (minutes)	1	2	3	4	5	6
Volume (10^6 cm^3)		8				

(ii) Draw a graph to show the total volume of oil on the water over the first 6 minutes.



(iii) Write an equation for $V(t)$, the volume of oil on the water, in cm^3, after t minutes.

(b) The spilled oil forms a circular oil slick **1 millimetre** thick.

(i) Write an equation for the volume of oil in the slick, in cm^3, when the radius is r cm.

(ii) Find the rate, in cm per minute, at which the radius of the oil slick is increasing when the radius is 50 m.

(c) Show that the area of water covered by the oil slick is increasing at a constant rate of 4×10^7 cm^2 per minute.

(d) The nearest land is 1 km from the point at which the oil-spill began. Find how long it will take for the oil slick to reach land. Give your answer correct to the nearest hour.

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Question 9 **(50 marks)**

The approximate length of the day in Galway, measured in hours from sunrise to sunset, may be calculated using the function

$$f(t) = 12{\cdot}25 + 4{\cdot}75\sin\left(\frac{2\pi}{365}t\right),$$

where t is the number of days after March 21st and $\left(\frac{2\pi}{365}t\right)$ is expressed in radians.

(a) Find the length of the day in Galway on June 5th (76 days after March 21st). Give your answer in hours and minutes, correct to the nearest minute.

(b) Find a date on which the length of the day in Galway is approximately 15 hours.

(c) Find $f'(t)$, the derivative of $f(t)$.

(d) Hence, or otherwise, find the length of the longest day in Galway.

(e) Use integration to find the average length of the day in Galway over the six months from March 21st to September 21st (184 days). Give your answer in hours and minutes, correct to the nearest minute.

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