25241

M 30

AN ROINN OIDEACHAIS AGUS EOLAÍOCHTA

LEAVING CERTIFICATE EXAMINATION, 1998

MATHEMATICS — HIGHER LEVEL
PAPER 2 (300 marks)

FRIDAY, 12 JUNE — MORNING, 9.30 to 12.00

Attempt **five** questions from Section **A** and **one** question from Section **B.**
Each question carries 50 marks.

Marks may be lost if necessary work is not clearly shown or if you do not indicate where a calculator has been used.

SECTION A

1. *(a)* $p(k, 2)$ and $q(-6, -k)$ are the end points of a diameter of a circle S with centre $(3, -5)$.

Find the value of k.

Verify that the radius length of S is $\sqrt{130}$.

(b) K is the circle with equation $x^2 + y^2 = 100$.

Show, by calculation, that the point $a(12, -9)$ lies outside K.

Find the equation of the line oa, where o is the origin.

Find the coordinates of the points where oa intersects K.

(c) A circle of radius length $\sqrt{20}$ contains the point $(-1, 3)$. Its centre lies on the line $x + y = 0$.

Find the equations of the two circles that satisfy these conditions.

OVER→

2. *(a)* *abcd* is a parallelogram where $\vec{a} = 2\vec{i} - 7\vec{j}$, $\vec{b} = -6\vec{i} - 11\vec{j}$ and $\vec{c} = -8\vec{i} + 4\vec{j}$.

Express $\vec{d}$ in terms of $\vec{i}$ and $\vec{j}$.

(b) $\vec{p} = 9\vec{i} - 5\vec{j}$, $\vec{q} = 5\vec{i} + 3\vec{j}$ and $\vec{s} = -5\vec{i} - \frac{9}{2}\vec{j}$.

Let $\vec{m} = \frac{1}{2}(\vec{p} + \vec{q})$ and $\vec{n} = \frac{2}{5}(\vec{sq})$.

(i) Express $\vec{m}$ and $\vec{n}$ in terms of $\vec{i}$ and $\vec{j}$.

(ii) Find the measure of the angle between $\vec{m}$ and $\vec{n}$.

(c) $\vec{x} = -3\vec{i} + 4\vec{j}$ and $\vec{y} = 5\vec{i} + 12\vec{j}$.

(i) Find $|\vec{x}|$ and $|\vec{y}|$.

(ii) If $\vec{r} = (1 - t)\vec{x} + t\vec{y}$, where $t = \dfrac{|\vec{x}|}{|\vec{x}| + |\vec{y}|}$,

express $\vec{r}$ in terms of $\vec{i}$ and $\vec{j}$.

(iii) If $k\left(\dfrac{\vec{x}}{|\vec{x}|} + \dfrac{\vec{y}}{|\vec{y}|}\right) = 18\vec{r}$,

find the value of k, $k \in \mathbf{N}$.

3. *(a)* The parametric equations $x = 3 - 4t$ and $y = 1 + 2t$ represent a line, where $t \in \mathbf{R}$.

Find the Cartesian equation of the line.

(b) Find the equation of the line *pq* where *p* has coordinates (7,–6) and *q* has coordinates (–3,2).
Find the point of intersection of *pq* and the line $2x - 3y + 1 = 0$.
Determine the ratio in which the line $2x - 3y + 1 = 0$ divides [*pq*].

(c) **(i)** The line M is $ax + by + c = 0$.

Prove that the perpendicular distance from the point (x_1, y_1) to the line M is given by

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}.$$

(ii) If p is the length of the perpendicular from the origin to the line $\frac{x}{a} + \frac{y}{b} = 1$,

prove that

$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}.$$

4. *(a)* Find the values of θ for which $\cos\theta = \frac{\sqrt{3}}{2}$, where $0° \leq \theta \leq 360°$.

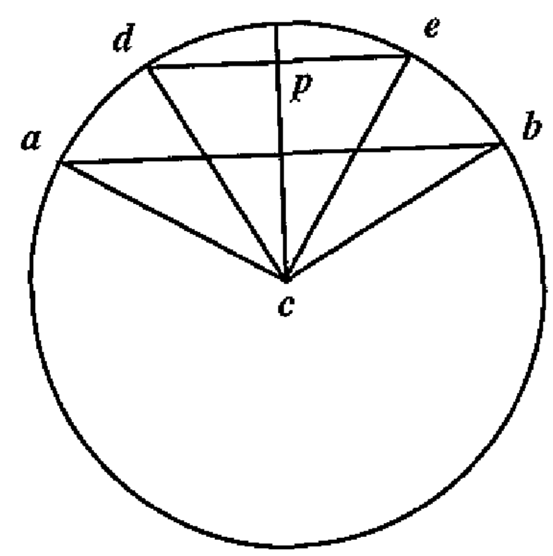
(b) Find the two solutions of the equation

$$4\sin^2 x - 3\cos x - 3 = 0,$$

where $0° \leq x \leq 180°$.

Give your answers correct to the nearest degree.

(c) $[ab]$ and $[de]$ are two parallel chords of a circle with centre c and radius length r.
$cp \perp de$, $|\angle acb| = 4\beta$ and $|\angle dce| = 2\beta$, where β is in radian measure, $\beta \neq 0$.



(i) If the area of the triangle acb = the area of triangle dce, show that $\beta = \frac{\pi}{6}$.

(ii) Calculate the value of r if

$$|ab|^2 + |de|^2 = 24$$

and give your answer as a surd.

5. *(a)* Express sinA in terms of t if

$$\tan A = \frac{t}{2}, \text{ where } t > 0 \text{ and } 0° < A < 90°.$$

(b) If $\tan A = \frac{1}{2}$, find tan2A without evaluating A, where A is an acute angle.

Express tanB in the form $\frac{a}{b}$, where $a, b \in \mathbf{N}_0$, given that

$$\tan(2A + B) = \frac{63}{16}.$$

(c) Express sin2A + sin2B as a product in sine and cosine.

If $A + B + C = 180°$, show that

$$\sin(A + B) = \sin C.$$

Hence, show that

$$\sin 2A + \sin 2B - \sin 2C = 4\cos A \cos B \sin C.$$

Note: $\cos(A + B) = -\cos C$.

6. *(a)* In how many ways can the letters of the word IRELAND be arranged if each letter is used exactly once in each arrangement?

In how many of these arrangements do the three vowels come together?

(b) Solve the difference equation

$$4u_{n+2} - 25u_{n+1} - 29u_n = 0, \quad \text{where } n \geq 0,$$

given that $u_0 = 0$ and $u_1 = 16.5$.

(c) On an unbiased die, the numbers 1, 3 and 4 are coloured red and the numbers 2, 5 and 6 are coloured black.

(i) The die is thrown once. Find the probability of getting an even number or a red number.

(ii) The die is thrown three times with the following outcome:

the second throw shows a red number and the sum of the numbers on the first and second throws is equal to the number on the third throw.

Find the probability of this outcome.

7. *(a)* If p is the mean of the numbers a, b, c, d express in terms of p and k the mean of the numbers $2a + k, 2b + k, 2c + k, 2d + k$.

(b) A classroom contains 15 desks which are arranged in rows.
The front row contains 3 desks.
15 students are seated at random in the classroom, 8 of whom are boys and 7 of whom are girls.
Each desk seats only one student.

What is the probability that

(i) three girls occupy the front row of desks?

(ii) there are more boys than girls seated in the front row?

(iii) there are two girls and one boy seated in the front row with the two girls seated next to each other?

(c) The numbers p, q, r have a mean $\bar{x}$ and standard deviation σ.

(i) Express $\bar{x}$ in terms of p, q and r.

(ii) Show that

$$\sigma^2 = \frac{1}{3}(p^2 + q^2 + r^2) - (\bar{x})^2.$$

SECTION B

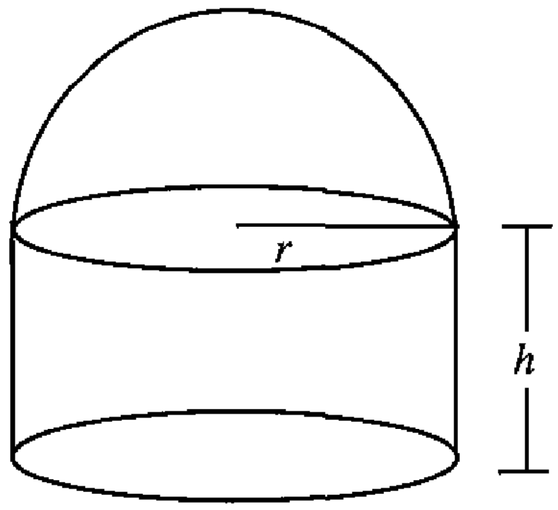
Answer ONE question from this section.

---

8. *(a)* Use the ratio test to show that $\sum_{n=1}^{\infty} \frac{n}{2^n}$ is convergent.

(b) Evaluate $\int_0^{\pi/2} x^2 \cos x \, dx$.

(c) A tank with a base is made from thin uniform metal. The tank standing on level ground is in the shape of an upright circular cylinder and hemispherical top with radius of length r metres. The height of the cylinder is h metres.



(i) If the total surface area of the tank is $45\pi \text{ m}^2$, express h in terms of r.

(ii) Find the value of h and of r for which the tank has maximum volume.

9. *(a)* A student takes two tests, A and B. The probability of passing in test A is $\frac{1}{2}$ and the probability of passing in test B is $\frac{2}{3}$. The probability of passing in both tests is $\frac{1}{3}$. Find the probability of passing in at least one of the two tests.

(b) A car manufacturing company tested a random sample of 150 cars of the same model to estimate the mean number of kilometres travelled per litre of petrol consumption for all cars of that model.

The sample mean of kilometres travelled per litre of petrol consumed was 13.52 and the standard deviation was 2.23.

Form a 95% confidence interval for the mean number of kilometres travelled per litre of petrol consumed for all cars of that model.

Give all calculations correct to two places of decimals.

(c) The lifetime of a particular type of electric bulb is normally distributed with a mean of 1500 hours and a standard deviation of 120 hours.

If 140 bulbs are purchased, how many can be expected to have a lifetime between 1400 hours and 1730 hours inclusive?

Give your answer correct to the nearest whole number.

10. *(a)* Verify that $\{1, i, -1, -i\}$ forms a group under multiplication, which is assumed associative and where $i = \sqrt{-1}$.

(b) The set $\{1, 2, 4, 5, 7, 8\}$ mod 9 is a group under multiplication, which is assumed associative.

(i) Write down the inverse of each element.

(ii) State the order of the elements 4, 7 and 8.

(iii) Show that there is a subgroup of order 2 and a subgroup of order 3.

(c) $G,\circ$ and $H,*$ are two groups with identities e_1 and e_2, respectively.

If $\phi : G \rightarrow H$ is an isomorphism, show that

(i) $\phi(e_1) = e_2$

(ii) $\phi(x^{-1}) = [\phi(x)]^{-1}$, for all $x \in G$.

11. *(a)* Find the equation of the ellipse with one focus at (3, 0) and where (–5, 0) and (5, 0) are the coordinates of the end points of the major axis.

(b) A triangle pqr is mapped onto the triangle $p'q'r'$ under a similarity transformation g.

If h is the orthocentre of the triangle pqr, prove that $g(h)$ is the orthocentre of triangle $p'q'r'$.

(c) f is the transformation $(x, y) \rightarrow (x', y')$ where

$$x' = ax$$
$$y' = by, \text{ for } a > b > 0.$$

C is the circle $x^2 + y^2 = 1$.

Show that $f(\text{C})$ is an ellipse E.

Hence show that the locus of midpoints of parallel chords of the ellipse E is a diameter (less its end-points) of E.