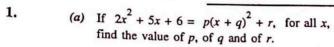
LEAVING CERTIFICATE EXAMINATION, 1995 24529

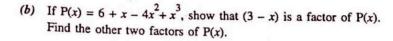
MATHEMATICS - HIGHER LEVEL - PAPER I (300 marks)

THURSDAY, 8 JUNE - MORNING, 9.30 to 12.00

Attempt SIX QUESTIONS (50 marks each)

Marks may be lost if all your work is not clearly shown? APR or if you do not indicate where a calculator has been used.





(c) (i) Solve the inequality
$$\frac{5-x}{x-2} < 1, x \neq 2 \text{ and } x \in \mathbb{R}.$$

$$y \ge 0$$
 and $y \le 3 - |x|$.

$$y = 2x$$

$$3x^2 - 2xy + y^2 = 9.$$

(b) (i) If
$$\alpha$$
 and β are the roots of the equation $x^2 - px + q = 0$, show that

$$(\alpha - \beta)^2 = p^2 - 4q.$$

(ii) Let
$$f(x) = \left(\frac{b^n - a^n}{b - a}\right)x + ab\left(\frac{a^{n-1} - b^{n-1}}{b - a}\right)$$
, for $a \neq b$.

Show that $f(a) = a^n$.

(c) For a fixed k > 0, let

$$f(x) = x^3 - k^2 x, x \in \mathbb{R}.$$

For $p \neq q$, divide f(q) - f(p) by q - p.

Prove that if $0 \le p < q \le \frac{k}{\sqrt{3}}$, then f(q) < f(p), whereas

if
$$\frac{k}{\sqrt{3}} \le p < q$$
, then $f(q) > f(p)$.

3. (a) The complex number u = 4 + 3i, where $i^2 = -1$. Find the complex number v = p + qi, $p, q \in \mathbb{R}$, where

$$uv = 10 - 5i.$$

(b) Using De Moivre's theorem, prove that

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta.$$

(c) Let
$$M = \begin{pmatrix} 1 & -2 \\ 3 & 1 \end{pmatrix}$$
 and $A = \begin{pmatrix} 7 & 10 \\ 21 & 23 \end{pmatrix}$.

- (i) Simplify $M^{-1}A$.
- (ii) If MB = 2M + A, express B in matrix form.
- (iii) Express the simultaneous equations

$$\begin{aligned}
x - 2y &= 3 \\
3x + y &= -1
\end{aligned}$$

in matrix form and use matrix methods to solve them.

- 4. Solve $\binom{n+2}{2} = 36$, for $n \in \mathbb{N}$.
 - (b) Show that $\frac{2}{n(n+2)} = \frac{1}{n} \frac{1}{n+2}$.

If
$$u_n = \frac{2}{n(n+2)}$$
, find $\sum_{n=1}^{\infty} u_n$.

(c) (i) Write the first three terms in ascending powers of b and the general term in the binomial expansion of $(a + b)^n$ for $n \in \mathbb{Z}^+$.

If h is a constant and hx^3y^4 is a term in the expansion of $(2x + 5y^2)^n$, find the value of n and the value of h.

(ii) If $S_n = \frac{1}{n}\sqrt{1+2+3+\cdots+n}$, find $\lim_{n\to\infty} S_n$.

- 5. (a) The first three terms of an arithmetic sequence are 6, -9 and x. The first three terms of a geometric sequence are -9, x and y. Find the value of x and the value of y.
 - (b) (i) Find the value of x in

$$\log_2(x+2) + \log_2(x-2) = 5.$$

- (ii) If $2^x + 2^{1-x} 3 = 0$, solve for x.
- (c) Prove, using the method of induction, that if r > 0,

$$\frac{1}{(1+r)^n} \leq \frac{1}{1+nr}, \text{ for all } n \geq 1.$$

Deduce that $\lim_{n \to \infty} x^n = 0$, if 0 < x < 1.

6. (a) Find the derivative of the functions

(i)
$$(4x-1)^3$$

(ii)
$$\frac{x}{x^2 + x + 1}$$
.

(b) Find the derivative of the functions

(i)
$$x^2 \log_e (2x+1)$$
, for $x > -\frac{1}{2}$

(ii)
$$\tan^{-1}\left(\frac{1}{x}\right)$$
, for $x \neq 0$.

(c) (i) The concentration C of an antibiotic in the bloodstream after a time of t hours is given by

$$C = \frac{5t}{1 + \left(\frac{t}{k}\right)^2} \quad \text{units} ,$$

where k > 0.

If the maximum concentration is reached at t = 6 hours, find the value of k.

(ii) Prove, from first principles, the product rule

$$\frac{d}{dx} (uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

where u = u(x) and v = v(x).

7. (a) Find the slope of the tangent to the curve

$$4x^2 + 9y^2 = 40$$

at the point (1, 2).

(b) x_n is the *n*-th approximation to the positive root of $x^2 - 2 = 0$ and x_{n+1} is the next approximation.

Using the Newton-Raphson method, $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$, show that

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right).$$

If $x_0 = 1$, find x_2 correct to three places of decimals.

(c) Let $x = \frac{1}{2} (e^{y} - e^{-y})$.

Show that

$$y = \log_e (x + \sqrt{x^2 + 1}).$$

Hence, show that $\frac{dy}{dx}$ can be expressed in the form $\frac{p}{(1+x^2)^q}$, $p, q \in \mathbf{R}$.

- 8.
- (a) Find (i) $\int (1+3x^2)dx$, (ii) $\int \cos 2x dx$.
- (b) Determine the area enclosed by the curve $y = x^2 + 1$ and the line y = 5.
- (c) (i) Evaluate

$$\int_{0}^{36} \cos^{2} 3\theta d\theta$$

and

$$\int_{0}^{4} \frac{dx}{\sqrt{x} (1 + \sqrt{x})}$$

(ii) Derive, by integration methods, the volume of a cone of vertical height h and base radius length r.