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M. 30

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LEAVING CERTIFICATE EXAMINATION, 1994

MATHEMATICS - HIGHER LEVEL

PAPER 2 (300 marks)

FRIDAY, 10 JUNE – MORNING, 9·30 to 12·00

Attempt five questions from Section A and one question from Section B. Each question carries 50 marks.

Marks may be lost if necessary work is not shown or if you do not indicate where a calculator has been used.

SECTION A

1. (a) Find the Cartesian equation of the circle

$$x = -2 + \frac{1}{2}\sin\theta, \quad y = 1 + \frac{1}{2}\cos\theta.$$

(b) $p\,(3, -1)$, $q\,(3, -4)$ and $r\,(x, y)$ are points such that $|pr| = 2\,|qr|$.

Verify that r is on a circle.

Find the centre and radius-length of the circle.

(c) Verify that the circles S_1 and S_2 touch externally if

$$S_1 : x^2 + y^2 - 2x + 4y - 8 = 0$$

$$S_2 : x^2 + y^2 + 6x - 8y + 12 = 0.$$

Verify that $8x - 12y + 20 = 0$ is a tangent to each of the circles at their point of contact.

2. (a) (i) r is an internal point of $[pq]$ such that $|pr| = \frac{2}{3}\,|pq|$.

Express $\vec{r}$ in terms of $\vec{p}$ and $\vec{q}$.

(ii) If $\vec{p}$ is $\vec{i} - \vec{j}$ and $\vec{q}$ is $4\vec{i} + 2\vec{j}$, express $\vec{r}$ in term of $\vec{i}$ and $\vec{j}$, these being the unit vectors along the horizontal and vertical directions, respectively.

Show that the acute angle between $\vec{r}$ and $\vec{pr}$ is given by

$$\cos^{-1}\frac{1}{2\sqrt{5}}.$$

[redacted]

(b) If $\vec{r}^{\perp} = -y\vec{i} + x\vec{j}$ where $\vec{r} = x\vec{i} + y\vec{j}$, show which of the following are always true

(i) $(\vec{r} + \vec{s})^{\perp} = \vec{r}^{\perp} + \vec{s}^{\perp}$

(ii) $(\vec{r}^{\perp})^{\perp} = \vec{r}$

(iii) $(k\vec{r})^{\perp} = k(\vec{r}^{\perp})$

(iv) $\vec{r}^{\perp}.\ \vec{s} = \vec{r}.\ \vec{s}^{\perp}$.

OVER →

3. For the linear transformation

$$f : (x, y) \to (x', y'), \text{ where}$$

$$x' = x + y$$

$$y' = 2x - y,$$

(i) express x and y in terms of x' and y'.

(ii) find the image of (1, 0) and the image of (0,1).

(iii) find f (L), the equation of the image of L : $3x - 4y + 6 = 0$ under the transformation f.

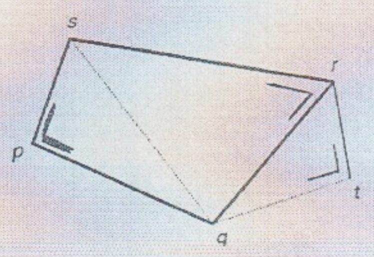
(iv) if M is a line through the origin perpendicular to L, investigate if $f\,(\mathrm{M}) \perp f\,(\mathrm{L})$.

4. *(a)* Show that $\dfrac{2\tan A}{1 + \tan^2 A} = \sin 2A$.

(b) Find the size of the greatest angle of the triangle which has sides of length 3, 5 and 7.

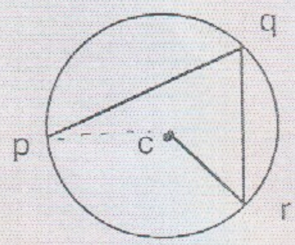
(c) A plane piece of wood $pqrs$ is propped at an angle of 60° to the horizontal by a vertical rod rt.

If $|\angle spq| = |\angle qrs| = 90°$ and $|\angle sqr| = 65°$, find $|\angle pqs|$ correct to the nearest degree given that $|ps| = |rt|$.



5. *(a)* p, q and r are points of a circle, centre c. The radius of the circle measures 3 cm.

$|\angle pqr| = 60°$. Find the length of the minor arc pr.



(b) Find x if $\dfrac{1}{\sqrt{3}} \sin x = \cos \dfrac{x}{2}$, where $0 \le x \le 2\pi$.

(c) A vertical flagpole stands on horizontal ground. The angle of elevation of the top of the pole from a certain point on the ground is θ. From a point on the ground 10 metres closer to the pole the angle of elevation is β. Show that the height of the pole is

$$\frac{10 \sin\theta \sin\beta}{\sin(\beta - \theta)}$$

6. (a) Complete the following frequency distribution table.

Class interval	2-----4	4-----8	8-----14	14-----18
Mid interval, x_r	3			
Frequency, f_r	2	6	9	3

Calculate the mean of the frequency distribution.

For each x_r, calculate $| d_r |$, where d_r is the deviation of x_r from the mean.

(b) A bag contains twelve coloured marbles; three red, five green and four white. What is the probability that in a single random selection the result will be a red *or* a green marble?

A rule is made so that when marbles of the same colour are drawn in consecutive selections, the first is returned to the bag and the second is replaced by a marble of one of the other two colours.

The first three marbles drawn were white.
What is the probability that the fourth marble drawn is either red or green?

(c) An unbiased coin is tossed five times. What is the probability that the outcome is

(i) two heads and three tails?

(ii) At least two heads?

7. (a) In how many ways can the letters of the word LEAVING be arranged if

(i) L is always at the beginning?

(ii) E and A are always side by side, as EA or AE?

(b) (i) Prove that if α and β are the roots of the quadratic equation

$$px^2 + qx + r = 0 \text{ and}$$

$$u_n = l\alpha^n + m\beta^n \text{ for all } n \text{ then}$$

$$pu_{n+2} + qu_{n+1} + ru_n = 0 \text{ for all } n.$$

(ii) Solve the difference equation

$$6u_{n+2} - 5u_{n+1} - 6u_n = 0$$

given $u_1 = 2$ and $u_0 = 3$.

OVER →

SECTION B

Do ONE question

8. *(a)* Use the ratio test to show that $\sum_{n=1}^{\infty} \frac{(n+1)}{2^n}$ is convergent.

(b) Evaluate $\int_0^{\pi} e^x \sin x \, dx$

(c) A plastic cylinder has one end open and the other end closed. The external surface area of the cylinder is 108π. Find its maximum volume. (Ignore the thickness of the plastic).

9. *(a)* An article is produced by either of two machines A and B. 60% of a certain consignment is produced by machine A. The probability that an article produced by machine A is defective is 0.05 while 10% of the production of machine B is defective. If an article is chosen at random from the consignment, what is the probability that it is defective.

(b) A random sample of 100 men in a certain country yields a mean height for the sample of 185 cm. The standard deviation for the country is 5 cm. Given that the heights of men are normally distributed in that country, find the 95% confidence interval for the mean height of the population.

(c) Of five balls in a bag, one bears the number 1, another the number 3, two others the number 4 and one the number 6. Two balls are drawn together. If an outcome is the product of the numbers on the two balls, write out the probability for each of the possible outcomes and evaluate the mean.