AN ROINN OIDEACHAIS **M. 29**

LEAVING CERTIFICATE EXAMINATION, 1994

MATHEMATICS - HIGHER LEVEL - PAPER I (300 marks)

21788

THURSDAY, 9 JUNE – MORNING, 9·30 to 12·00

Attempt **SIX QUESTIONS** (50 marks each)

Marks may be lost if all your work is not clearly shown or if you do not indicate where a calculator has been used.

1. *(a)* Solve the equation

$$2x - 1 = \sqrt{8x + 1}\,.$$

(b) Solve the simultaneous equations

$$3x + 5y - z = -3$$
$$2x + y - 3z = -9$$
$$x + 3y + 2z = 7$$

(c) $x^2 + ax + b$ is a factor of the polynomial $x^3 + qx^2 + rx + s$.

Prove that $r - b = a(q - a)$ and $s = b(q - a)$.

2. *(a)* Solve the simultaneous equations

$$y = x - 3$$

$$x^2 - 3y^2 = 13$$

(b) The roots of the equation $2x^2 + 6x + 3 = 0$ are α and β.

Show that $\alpha^2 + \beta^2 = 6$.

The roots of $2x^2 + px + q = 0$ are $2\alpha + \beta$ and $\alpha + 2\beta$.

Find the value of p and the value of q.

(c) If for all integers n,

$$u_n = (n - 20)\, 2^n\,,$$

verify that

$$u_{n+2} - 4u_{n+1} + 4u_n = 0.$$

For what values of $n > 0$ is $u_{n+1} > 3u_n$?

OVER ➸

3. (a) If the matrix $M = \begin{bmatrix} 3 & -1 \\ 4 & -2 \end{bmatrix}$, find $M^2 - 3M$.

(b) Express in the form $p + iq$, where $p, q \in \mathbf{R}$ and $i^2 = -1$,

(i) $\dfrac{5 - 2i}{4 + 3i}$

(ii) $(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})^5$

(c) Determine real numbers s and t so that

$$(s + it)^2 = -3 + 4i.$$

Hence determine the two roots of the equation

$$z^2 - (4 - 2i)z + 6 - 8i = 0.$$

4. (a) Find the sum of the infinite geometric series

$$1 + \frac{3}{2x + 1} + \left(\frac{3}{2x + 1}\right)^2 + \bullet \bullet \bullet + \left(\frac{3}{2x + 1}\right)^{n-1} + \bullet \bullet \bullet, \text{ where } x > 1.$$

(b) The sum of the first n terms of the series $u_1 + u_2 + u_3 + \bullet \bullet \bullet + u_n$ is given by $n(n - 1)$.

(i) Express u_n in terms of n.

(ii) If u_n, u_{n+1}, u_{n+2} and u_{n+3} are four terms of the series, calculate the value of

$$(u_{n+3}^2 - u_{n+1}^2) - (u_{n+2}^2 - u_n^2).$$

(c) Show that

$$(ax + by)^2 \leq (a^2 + b^2)(x^2 + y^2)$$

for all $a, b, x, y \in \mathbf{R}$.

5. (a) Solve the equation

$$\log_{10}(17 - 3x) + \log_{10} x = 1.$$

(b) Using mathematical induction show that

$$2^{3n-1} + 3$$

is divisible by 7 for all $n \in \mathbb{N}_0$.

(c) If $f(x) = \sum_{n=1}^{\infty} nx^{n-1}$, for $-1 < x < 1$,

show that

$$x f(x) = \sum_{r=2}^{\infty} rx^{r-1} - \sum_{r=2}^{\infty} x^{r-1}.$$

Hence, find the value of $f(x)$.

6. (a) Find the derivative of the functions

(i) $\dfrac{x^2 - 1}{x^2 + 1}$

(ii) $\sqrt{1 + 3x}$

(b) (i) Find the derivative of $\sin^{-1} 3x$.

(ii) Let $x + y = 12$, where $x, y > 0$.

If $A = x^2 + y^2 + 3xy$, write A as a quadratic in x.

Calculate the maximum value of A.

(c) Let $f(x) = \dfrac{x}{x - 3}$, $x \neq 3$ and $x \in \mathbb{R}$.

(i) Show that the curve $f(x)$ has no points of inflection.

(ii) Find the equations of the asymptotes of the curve $f(x)$.

(iii) Draw a sketch of the curve $f(x)$.

(iv) Find how x_1 and x_2 are related if the tangents at $(x_1, f(x_1))$ and $(x_2, f(x_2))$ are parallel.

OVER ➔

7.

(a) Differentiate from first principles $\frac{1}{x}$ with respect to x.

(b) *(i)* Find the derivative of $\log_e (1 + \tan x)$.

(ii) The path of a football is given by the equation

$$y = x - \frac{x^2}{40}, \quad x \geq 0.$$

If $\frac{dx}{dt} = 10\sqrt{2}$ for all t, find $\frac{dy}{dt}$ when $x = 10$.

(c) Given that $x = e^{\theta}\cos\theta$ and $y = e^{\theta}\sin\theta$, where $\frac{-3\pi}{4} < \theta < \frac{\pi}{4}$, show that

(i) $$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = 2e^{2\theta}$$

(ii) $$\frac{dy}{dx} = \tan\left(\theta + \frac{\pi}{4}\right)$$

Note: See Tables, p. 9.

8.

(a) Find $\int (4x - 3)dx$ and $\int \sin 3x dx$.

(b) Find the area of the bounded region enclosed by the curve $y = \frac{2x}{x^2 + 1}$, the x axis, the line $x = 1$ and the line $x = 2\sqrt{2}$.

(c) *(i)* Evaluate

$$\int_0^1 \frac{x^2 - 16}{2x + 8} dx \quad \underline{\text{or}} \quad \int_0^{\frac{1}{3}} \frac{1}{1 + 9x^2} dx.$$

(ii) If $a > 0$ and

$$\int_0^b \frac{1}{1 + x} dx = \tfrac{1}{2}\int_0^a \frac{1}{1 + x} dx,$$

express b in terms of a.