David Malone

# AN ROINN OIDEACHAIS

M. 30

## LEAVING CERTIFICATE EXAMINATION, 1992

### MATHEMATICS - HIGHER LEVEL - PAPER II (300 marks)

14514

FRIDAY, 12 JUNE - MORNING, 9.30 to 12.00

Attempt **QUESTION 1** (100 marks) and **FOUR** other questions (50 marks each)

**Marks may be lost if necessary work is not clearly shown or if you do not indicate where a calculator has been used.**

**1.**

10 *(i)* Solve $\sqrt{x} - \dfrac{12}{\sqrt{x}} = 1, \quad x \in \mathbf{R}.$

$4 - 3 = 1$

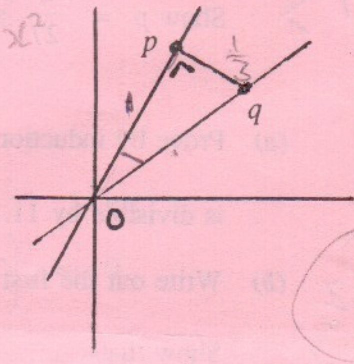
*(ii)* When $x = \dfrac{4}{1 + t^2}, \quad y = \dfrac{4t}{1 + t^2},$

write $y^2$ in terms of $x$.

3

$\frac{4-x}{x}$ $\quad 4x - x^2$

*(iii)* The diagram shows two lines through the origin. Their slopes are 2 and 1. If $|op| = 1$ calculate $|pq|$.

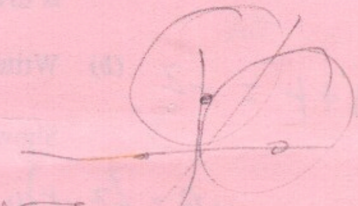


$\frac{1}{\sqrt{2}}$

*(iv)* Show that the circles

$$(x - 1)^2 + y^2 = 72$$
$$x^2 + (y - 1)^2 = 50$$

touch each other.



$\boxed{800}$

10 *(v)* Solve $\dfrac{2x - 1}{x + 1} \leq 1, \quad x \in \mathbf{R}, \quad x \neq -1.$

10 *(vi)* Prove that $\dbinom{n}{r} + \dbinom{n}{r-1} = \dbinom{n+1}{r}.$

*(vii)* The matrix for the axial symmetry in a line through the origin is

$$\begin{pmatrix} 0{\cdot}6 & 0{\cdot}8 \\ 0{\cdot}8 & -0{\cdot}6 \end{pmatrix}.$$

7

Find the equation of the line.

7 *(viii)* Find the matrix of the rotation which maps the X axis on to the line $3x + y = 0$.

10 *(ix)* Find the period of

$$\cos 2x \cos 5x.$$

$2\pi$ 10

10 *(x)* $G = \{1, 2, 3, 4, 5, 6\}$ is a group under multiplication mod 7.
Write out the subgroup of order 3. 10

**OR**

*(x)* The focal chord perpendicular to the axis of the parabola $y^2 = 8x$ cuts the parabola at $p$ and $q$. Write the equation of the tangent at either $p$ or $q$.

OVER→

**2.** *(a)* Given that a solution exists to the simultaneous equations

$$6x - 4y - z = \sqrt{3}$$
$$3x - 2y = r\sqrt{3}$$
$$9x - 6y - 2z = r^2\sqrt{3}$$

find a value for $r$, $r \neq 1$.

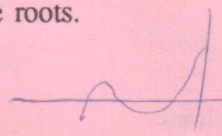
Write one solution to the set of equations.

*(b)* One root of the equation

$$3x^3 + 27px^2 + 2x + 3p = 0, \quad p > 0,$$

is the sum of the other two roots.

Show $p = \frac{4}{27}$ and evaluate the roots.



$\alpha + \beta + \gamma = -\frac{4}{3}$

$3x^3 + 4x^2 + 2x + \frac{4}{9} = 0$

$(3x+1)(x+1)$

$x = -1$

$x = -\frac{1}{3}$

**3.** *(a)* Prove by induction that

$$9.4^{2n} - 5^{n-1}$$

is divisible by 11.

*(b)* Write out the first 3 and the last three terms of the binomial expansion of $(1 - x)^n$.

$\beta + \gamma = -\frac{2}{3}$

Show that

$\beta\gamma = -\frac{4}{27}$

(i)

$$\sum_{r=0}^{n} \binom{n}{r} = 2^n$$

(ii) the sum of the coefficients of the odd terms is equal to the sum of the coefficients of the even terms.

Evaluate in the form $a^b$, $\displaystyle\sum_{r=0}^{10} \binom{20}{2r}$.

In a set containing 4 elements, how many subsets are there which contain 0, 2, 4 elements (i.e. an even number of elements) ?

S is a set # S = 20. How many subsets of S contain an even number of elements ?

$(2x + 3y)(x + 4y)$

**4.** The pair of lines

$$2x^2 + 11xy + 12y^2 - 10x - 25y + 12 = 0$$

form two sides of a triangle. One vertex of the triangle is $(5, -2)$ and another vertex is also in the fourth quadrant.

Find the vertices, if the area of the triangle is 3 .

**5.** Find the length of the tangents from $p\ (-4,\ 0)$ to the circle

$$x^2 + y^2 - 4x - 8y - 30 = 0 .$$

Find the equations of the tangents to the circle from $p$.

The line joining the centre of the circle to the point of tangency $t$ cuts the Xaxis inside the circle at $q$ .

Find the *(i)* coordinates of $t$
*(ii)* $|qt|$ .

**6.** *(a)* *(i)* $$M = \begin{pmatrix} 7 & 4 \\ 4 & 1 \end{pmatrix} .$$

Find the equations of the lines

$$M \begin{pmatrix} x \\ y \end{pmatrix} = 9 \begin{pmatrix} x \\ y \end{pmatrix} ; \quad M \begin{pmatrix} x \\ y \end{pmatrix} = -1 \begin{pmatrix} x \\ y \end{pmatrix} .$$

50

Sketch these lines.

*(ii)* Sketch also the pair of lines in

$$( x \ \ y ) \begin{pmatrix} 7 & 4 \\ 4 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

Prove that the pair of lines in (i) are axes of symmetry of the lines in (ii) .

*(b)* Find the condition that $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

has a solution when $(x, y) \neq (0, 0)$ .

Hence or otherwise show that there is a solution to

$$\begin{pmatrix} a & 1-b \\ 1-a & b \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

for all $a, b$, where $(x, y) \neq (0, 0, )$.

**7.** *(a)* Show that

$$\sqrt{\frac{1 + \cos\theta}{1 - \cos\theta}} = \frac{\tan\theta}{\sec\theta - 1} , \quad 0 < \theta < \frac{\pi}{2} .$$

*(b)* $u\,v\,x\,y\,z$ is a regular pentagon with side of length 1.

Show that $|xz| = 2\cos 36°$ .

By drawing perpendiculars from $u$ and $v$ to $xz$ , or otherwise, show that

$$|xz| = 1 + 2\sin 18°$$

Hence, write $\sin 18°$ in the simplest surd form.

OVER→

**8.** $G = \{1, 3, 5, 7\} \bmod 8$; $H = \{1, 5, 12, 8\} \bmod 13$.

Show $G$ and $H$ are groups under multiplication.

Is there an isomorphism $f: G \to H$ ?

Support your answer by investigating

$$f(x \cdot x) = f(x) \cdot f(x) \text{ for one } x \in G,\ x \neq 1.$$

Where $x, y \in G$ and $u, v, \in H$, the set of all couples of the type $(x, u)$ is formed and an operation $*$ is defined by

$$(x, u) * (y, v) = (x \cdot y,\ u \cdot v).$$

Evaluate $(3, 8) * (7, 12)$ and show that the operation $*$ is commutative for this pair.

Write the identity for $*$ and the inverse of $(3, 8)$ under the operation.

Show that $J_* = \{(x, u) \mid x \in G, u \in H\}_*$ is a group.

**OR**

**8.** Write the equation of the tangent to the parabola $y^2 = 4x$ at a point $(t_1^2, 2t_1)$.

Write the equation of the perpendicular to the tangent at the point $(t_1^2, 2t_1)$.

Write the equation of the tangent and the perpendicular to the tangent at $(t_2^2, 2t_2)$ on the parabola.

Show that the two perpendiculars intersect at

$$(t_1^2 + t_1 t_2 + t_2^2 + 2,\ -t_1^2 t_2 - t_1 t_2^2).$$

If these two tangents are perpendicular, show that $t_1 t_2 = -1$ and deduce that the locus of the above point of intersection is another parabola with the X axis as its axis of symmetry.