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LEAVING CERTIFICATE EXAMINATION, 1990

MATHEMATICS – HIGHER LEVEL – PAPER I (300 marks)

THURSDAY, 7 JUNE – MORNING, 9.30 to 12.00

Attempt **QUESTION 1** (100 marks) and **FOUR** other questions (50 marks each)

Marks may be lost if all your work is not clearly shown or if you have not indicated where a calculator has been used

1. (i) Solve the equation $(x + \frac{1}{x})^2 = 4$.

(ii) Use a graph, or otherwise, to find the range of values of x for which $\frac{x+2}{x-3} > 1$, for $x \in \mathbf{R}$, and $x \neq 3$.

(iii) Find $\tan\theta$, where θ is the obtuse angle between the lines $3x - 2y + 1 = 0$ and $3x + 2y - 1 = 0$.

(iv) Find the slope of the common chord of the circles $x^2 + y^2 + 2x - 3y + 1 = 0$, $x^2 + y^2 - x + 2y - 1 = 0$.

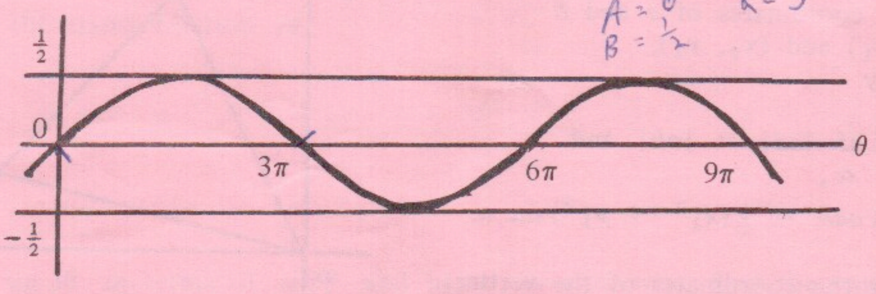
(v) Evaluate
$$\begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}^6$$

(vi) Find the equation of the image of the line $x = 1 - t$, $y = 1 + t$ for $t \in \mathbf{R}$ under the translation $(0, 0) \to (2, 1)$ and express your answer in the form $ax + by + c = 0$.

(vii) In the quadratic form
$$(x \;\; y)\begin{pmatrix} 3 & -1 \\ -1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 14$$
find y when $x = 2$.

(viii) Prove that the graph of $x \to \cos x$ is its own image under the axial symmetry of the plane in the line $x = \pi$.

(ix)



This is a graph of the function $\theta \to a + b\sin k\theta$, $b > 0$. Find the values of a, b, k.

(x) $\{1, a, b\}$ is a group under the operation $*$ where the identity element is 1. Say why $a * a = 1$.

OR (x) Find the equation of the image of the parabola $y^2 = 4x$ under the translation $(0, 0) \to (3, 2)$.

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2. If α, β, γ are the roots of the equation

$$ax^3 + bx^2 + cx + d = 0,$$

write down a cubic equation which has

$$7\alpha - 6, \quad 7\beta - 6, \quad 7\gamma - 6$$

as roots.

Hence, or otherwise, find the only rational root of the equation

$$7x^3 + x^2 + x - 6 = 0$$

in the form $\frac{p}{q}$.

3. (a) Show by induction, or otherwise, that

$$n(n + 1)(2n + 1) \quad \text{for} \quad n \in N_0$$

is divisible by 6.

(b) (i) Write out in ascending powers of x the first five terms of the expansion of $(1 + x)^{-1}$ and state the range of values of x for which the expansion is valid.

(ii) Noting that

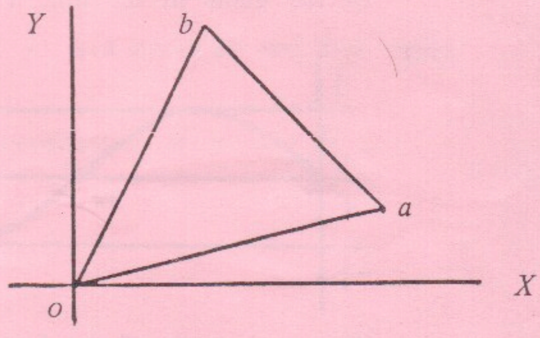
$$\int_0^x \frac{dt}{1 + t} = \log(1 + x), \quad 0 < x < 1$$

write the first five terms of the expansion of $\log(1 + x)$.

4. (i) Prove that the area of the Δoab is

$$\tfrac{1}{2}(x_1 y_2 - x_2 y_1)$$

where the coordinates of a and b are (x_1, y_1) and (x_2, y_2), respectively.



(ii) Prove that if $|oa| = |ob|$ and $|\angle aob| = \alpha$,
area of $\Delta oab = \frac{1}{2}(x_1^2 + y_1^2)\sin\alpha$.

(iii) Given that the coordinates of the vertices of a triangle are rational numbers, prove that the triangle cannot be equilateral.
Show also that the triangle could be isosceles and give an example of one such triangle.

5. Write down the coordinates of the centre of the circle

$$S : x^2 + y^2 - 4x + 2y - 20 = 0$$

and find the length of its radius.

Verify that $p(-1, 3)$ is a point of the circle.

The tangent, T, to the circle at p cuts the X axis at q. Find the coordinates of q.

K is a line through q perpendicular to T. Verify that K is also a tangent to the circle and find the coordinates of t, its point of contact.

Calculate the ratio

Area of S : area of circle on $[pt]$ as diameter.

6. Let

$$M = \begin{pmatrix} a & k \\ k & -a \end{pmatrix}.$$

If $M^{-1} = M$, prove that $a^2 + k^2 = 1$ and then write out the matrix M given that $a = \frac{3}{5}$ and $k > 0$.

Express the vector $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ as a linear combination of the two vectors $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ and hence find r and s for which

$$M\begin{pmatrix} 3 \\ -1 \end{pmatrix} = r\begin{pmatrix} 2 \\ 1 \end{pmatrix} + s\begin{pmatrix} 1 \\ -2 \end{pmatrix}.$$

Give a geometrical interpretation of M and hence, or otherwise, find the equation of the image of the line

$$x - 2y = 0$$

under the transformation M.

7. Write $\sin 3\theta$ in terms of $\sin \theta$ and hence, or otherwise, find the values of θ in the range $0 \leqslant \theta \leqslant 2\pi$ for which

$$f(\theta) = \sin 3\theta + \cos 2\theta - \sin \theta = 1.$$

Use the tables on Page 9 to express $\sin 3\theta - \sin \theta$ as a product and hence find the values of θ in the range $0 \leqslant \theta \leqslant 2\pi$ for which

$$f(\theta) = \sin 3\theta + \cos 2\theta - \sin \theta = 0.$$

A function g is said to be odd if $g(-x) = -g(x)$ for all x.
Investigate if f is odd.

8. (a) (i) Show that

$$A = \left\{ 1, \ \frac{1+i}{1-i}, \ \frac{1-i}{1+i}, \ -1 \right\}$$

is a group under multiplication where $i = \sqrt{-1}$.

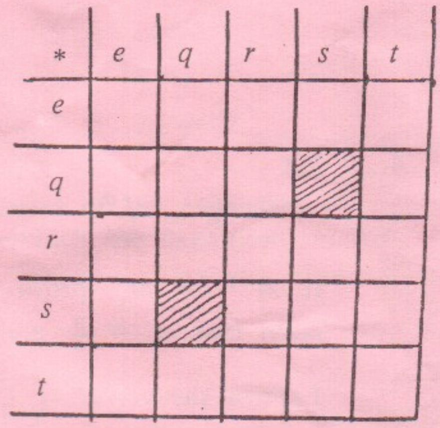
(ii) Let the set

$$C = \{1, a, a^2, a^3\}$$

be a group of order 4 under multiplication where 1 is the identity.
Write out one isomorphism $C, \times \rightarrow A, \times$.

(b) Explain the statement:
"a and b are inverse elements of a group $G, *$."

The set $\{e. \ q. \ r. \ s. \ t\}$ is a group under $*$ where e is the identity. If e occupies one of the shaded squares in the Cayley table, say why e also occupies the other.



A set S is a group under the operation $*$ where e is the identity. If S contains an even number of elements, prove that there is at least one element $k \in S$, where $k \neq e$ for which $k * k = e$.

OR 8. v is the vertex of the parabola $y^2 = 2x$. p and q are points of the parabola such that $vp \perp vq$. If m is the slope of vp, find in terms of m the coordinates of k, the midpoint of $[pq]$.

Show that for different values of m the locus of k is a parabola and that each $[pq]$ contains the vertex of this parabola.

If the coordinates of k are $(\frac{17}{4}, \frac{3}{2})$, find the coordinates of p and of q.