AN ROINN OIDEACHAIS

LEAVING CERTIFICATE EXAMINATION, 1985

MATHEMATICS - HIGHER LEVEL - PAPER I (300 marks)

THURSDAY, 13 JUNE - MORNING 9.30 to 12.00

Attempt **QUESTION 1** (100 marks) and **FOUR** other questions (50 marks each)

Marks may be lost if all your work is not clearly shown

1. (i) If $\frac{x+a}{y+b} = \frac{x}{y}$, express $\frac{a}{b}$ in terms of x and y.

(ii) Let T_4 be the fourth term in the expansion of

$$\left(1 + \frac{x}{n}\right)^n.$$

Find $\lim_{n \to \infty} T_4$.

(iii) From 7 colours, including Red, a firm chooses 4 to colour marbles. How many different coloured marbles have Red as one of its colours ?

(iv) $p(1, 2)$ and $q(0, 4)$ are two points. The point r is on pq, produced, and such that

$$|pq| : |qr| = 2 : 1.$$

Find the coordinates of r.

(v) A tangent to the circle $x^2 + y^2 = 8$ cuts the positive x-axis and the positive y-axis at points which are equidistant from the origin. Find the equation of this tangent.

(vi) Evaluate

$$(1 \quad i)\begin{pmatrix} 3 & 2-i \\ 2+i & 3 \end{pmatrix}\begin{pmatrix} 1 \\ -i \end{pmatrix}$$

where $i = \sqrt{-1}$.

(vii) $\frac{1}{3}\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}$ is the matrix of a linear transformation f. Find $f\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

If f is the parallel projection of the plane on a line L through the origin, find the equation of L.

(viii) Write down the image of the point $(x, \sin x)$ under the central symmetry in the origin and show that the graph of $x \to \sin x$ is its own image under this transformation.

(ix) If $\alpha\pi$ is the period of the function $x \to \cos(4x - 3)$, prove that $\alpha = \frac{1}{2}$.

(x) Solve the equation

$$X \,\Delta\, \{1, -i\} = \{1, i\}$$

in the set of all subsets of $\{1, i, -i, -1\}$, where Δ is symmetric difference.

OR

(x) Find the coordinates of the focus of the parabola

$$y^2 - 2y + 2x + 9 = 0$$

2. (a) Show by eliminating z that there is no unique solution of the equations

$$\begin{aligned} 5x + 3y - 2z &= 5 \\ 7x + 4y - 3z &= 6 \\ 3x + 2y - z &= 4 \end{aligned}$$

and find the maximum value of x for which y is non-negative.

(b) Show that the local maximum of the function

$$x \to 12x^3 - 27x - 27$$

is less than zero.

Verify that the equation

$$12x^3 - 27x - 27 = 0$$

has a root between 1 and 2 and say why there are no other real roots.

3. (a) Use a binomial expansion to evaluate

$$\sqrt{0{\cdot}998}$$

correct to 8 places of decimals.

Express $\dfrac{1}{1 + \sqrt{0{\cdot}998}}$ in the form $k(1 - \sqrt{0{\cdot}998})$ and hence, or otherwise find its value correct to 5 places of decimals.

(b) 4 identical dice are thrown. Find the total number of possible outcomes which do not contain a "6".

Note: The total number of ways of placing r identical objects in n boxes when more than one object can be placed in a box is $\binom{n + r - 1}{r}$.

4. Find the equation of the line through $p(2, 6)$ which cuts the y-axis at v, where $|\angle pvo| = 135°$, o being the origin.

If t is the point $(0, -4)$, show that 8 is the area of the Δpvt.

L is the line $x + y + 4 = 0$ and $q \in L$ where q, t, p are anti-clockwise. If

area of Δpqt = 5 (area of Δpvt)

find the coordinates of q.

Find the coordinates of the image of q under the axial symmetry in pv.

5. H is the circle $x^2 + y^2 + 8x - 10y + 32 = 0$.
Write down the coordinates of the centre of H and the length of its radius.
Draw a rough sketch of H.

K is a circle having its centre on the y-axis.
If $x - y + 6 = 0$ is the common chord of H and K, find the equation of K.

If f is the axial symmetry of the plane in the y-axis, find the coordinates of the points of intersection of $f(H)$ and $f(K)$.

6. (a) Write down the matrix of the rotation about the origin of angle θ.

If $\binom{u}{v}$ is the image of $\binom{h}{k}$ under this rotation, verify that $\binom{u}{v}$ and $\binom{h}{k}$ are equidistant from the origin.

Find the matrix of the rotation about the origin which maps the x-axis onto the line $y = 3x$. What is the equation of the image of the y-axis under this same rotation ?

(b) Let a linear transformation be defined by

$$A = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}.$$

Find $A\binom{1}{m}$

Find one value of m for which $y = mx$ is its own image under the transformation defined by A.

7. Write down the period and the range of each of the functions defined on **R**:

$$x \to \sin x \quad \text{and} \quad x \to (\sin x)^2.$$

Find the slope of the tangent to the graph of each function at the origin and sketch each function in the domain $-2\pi \leqslant x \leqslant 2\pi$.

Show that the function $x \to \dfrac{\sin x}{(\sin x)^2}$ has a local minimum at $\left(\frac{\pi}{2}, 1\right)$ and sketch the graph of the function in $0 < x < \pi$ indicating both asymptotes.

Also sketch the graph of the function in $-\pi < x < 0$.

8. Assuming that the composition of functions is associative show that the set

$$S = \left\{\begin{pmatrix} a & b & c \\ a & b & c \end{pmatrix}, \begin{pmatrix} a & b & c \\ b & c & a \end{pmatrix}, \begin{pmatrix} a & b & c \\ c & a & b \end{pmatrix}\right\}$$

is a commutative group under composition where $\begin{pmatrix} a & b & c \\ p & q & r \end{pmatrix}$ means $a \to p$, $b \to q$, $c \to r$. Name the identity element and the inverse of each element.

If a, b, c are the vertices of an equilateral triangle, what is the geometrical meaning of $\begin{pmatrix} a & b & c \\ b & c & a \end{pmatrix}$?

Find an element $K \in S$ such that

$$\begin{pmatrix} a & b & c \\ b & c & a \end{pmatrix}^{17} = K \circ \begin{pmatrix} a & b & c \\ c & a & b \end{pmatrix}.$$

Z_3, the set $\{\bar{0}, \bar{1}, \bar{2}\}$ of residue classes modulo 3 is a group under $+$ and the bijection f maps $Z_3 \to S$. Write out the couples of f and verify in each case that

$$f(x + y) = f(x) \circ f(y)$$

where $x, y \in Z_3$ and $x \neq y$.

OR

8. Write down the coordinates of the focus of the parabola $y^2 = 4ax$.

Verify that $p(at^2, 2at)$ and $q\left(\frac{a}{t^2}, -\frac{2a}{t}\right)$ are points of the parabola and prove that the three points p, q and the focus are collinear.

The tangent to the parabola at p meets the x-axis at r. Prove that $[pr]$ is bisected by the tangent to the parabola at its vertex.

The tangents to the parabola at p and q meet the y-axis at h and k. Find the equation of the circle having $[hk]$ as diameter and show that pq is a tangent to this circle at $(a, 0)$.