LEAVING CERTIFICATE EXAMINATION, 1982

MATHEMATICS - HIGHER LEVEL - PAPER I (300 marks)

1. (i) When $2x^3 + kx^2 - 48x - 24$ is divided by $2x + 1$, the remainder is 1. Find k.

(ii) Evaluate $\displaystyle\sum_{r=0}^{10} \binom{10}{r} \left(\frac{1}{4}\right)^r \left(\frac{3}{4}\right)^{10-r}.$

(iii) How many different arrangements, each containing 5 letters, can be made from the 9 letters

$$k,\ k,\ k,\ p,\ p,\ p,\ x,\ y,\ z$$

when each arrangement contains but one pair of same letters (k, p, k, x, y for example).

(iv) Find the equations of the two lines represented by

$$6x^2 - 7xy - 3y^2 - 18x + 38y - 24 = 0.$$

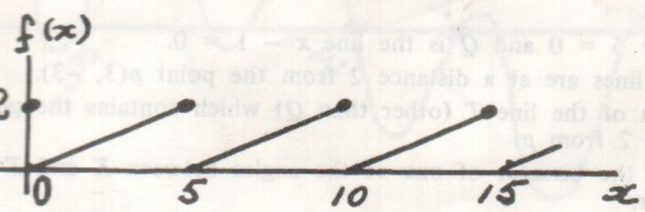
(v) $a(1, 2)$ and $b(-3, 4)$ are two given points. Find the equation of the locus of a point $p(x, y)$ such that $|\angle apb| = 90°$.

(vi) Evaluate

$$(1\ \ 2)\begin{pmatrix} 3 & 4 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

(vii) Find the matrix of the following composite map: the projection on the x-axis parallel to the y-axis after the rotation of 90° about the origin.

(viii)



$f(5n) = 2$
$n \in Z$

The diagram shows part of the graph of a periodic function $f : x \rightarrow f(x)$. Evaluate $f(213)$.

(ix) Prove $\dfrac{1 - \tan^2 A}{1 + \tan^2 A} = \cos 2A.$

$\cancel{\cos}$: $\cos^2 A - \sin^2 A = \dfrac{1 - \tan^2 A}{\cos^2 A}$

(x) Let k be the map $\begin{pmatrix} a & b & c & d \\ b & c & a & d \end{pmatrix}$ =

which means that $a \rightarrow b$, $b \rightarrow c$, $c \rightarrow a$, $d \rightarrow d$. Write k^{-1}, the inverse of k, in the form

$$\begin{pmatrix} a & b & c & d \\ ? & ? & ? & ? \end{pmatrix}$$

OR

(x) Establish that $t_1 y = x + at_1^{\,2}$ is the equation of the tangent to the parabola $x = at^2$, $y = 2at$ at the point $t = t_1$.

(Note: You may not assume $yy_1 = 2a(x + x_1)$ or $y = mx + \frac{a}{m}$).

2. (a) Given that

$$3x + 2y + 5z = -3$$
$$x + 2y + z = -5$$

express x in terms of z and y in terms of z and find a relation between p and q for which

$$px + qy$$

is independent of z.

(b) Find the roots of the equation

$$16x^3 - 36x^2 + 20x - 3 = 0$$

if one of the roots is twice another.

3. (a) Write out the first three terms of the expansion of

$$(x + \tfrac{1}{x})^{12}, \quad x > 0$$

in descending powers of x and express T_{r+1}, the general term, in the form $\binom{12}{r} x^k$.

If $T_6 > T_5$ when $x < \sqrt{t}$, find t.

(b) Show that $r\binom{n}{r} = n\binom{n-1}{r-1}$.

$r\frac{n!}{r!(n-r)!} = \frac{n(n-1)!}{(r-1)!(n-r)!} = n\binom{n-1}{r-1}$

Hence or otherwise, show that

$$n + 2\binom{n}{2} + 3\binom{n}{3} + \ldots\ldots + r\binom{n}{r} + \ldots\ldots + n = n.2^{n-1}.$$

$n\left(1 + \binom{n-1}{1} + \binom{n-1}{2} + \cdots\right) = n(1+1)^{n-1} = n.2^{n-1}$

4. K is the line $y + 5 = 0$ and Q is the line $x - 1 = 0$.

Verify that both lines are at a distance 2 from the point $p(3, -3)$.

Find the equation of the line T (other than Q) which contains the point $(1, -2)$ and which is also a distance 2 from p.

Say why p is on the bisector of one of the angles between K and T and find the equation of this bisector.

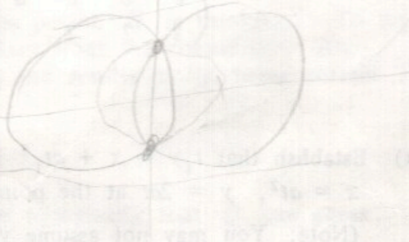
Let C be the circle having p as centre and K, Q, T as tangents. Verify that the area of C is greater than twice the area of the triangle formed by the three lines K, Q, T.

5. The line segment joining $(2, 4)$ and $(-6, 0)$ is the common chord of two circles. If the centre of each circle is at a distance of $\sqrt{20}$ from this common chord, verify that the radius of each circle is $\sqrt{40}$ and find the equation of each circle.

Assuming that each common tangent to the two circles is parallel to the line joining the centres of the two circles, find the equation of each common tangent.

Freq 5-11

lea 12th



$\int (\text{diff } x)^2 + (\text{diff } y)^2$

6. Define S_L, the axial symmetry of the plane in a given line L and say why S_L is equal to its inverse, S_L^{-1}.

Show that the matrix of the axial symmetry in a line which contains the origin and which makes an angle A with the x-axis is

$$\begin{pmatrix} \cos 2A & \sin 2A \\ \sin 2A & -\cos 2A \end{pmatrix}.$$

If K is the line $x - y\sqrt{3} = 0$, write down the matrix of S_K.

$y = x\sqrt{3} - \sqrt{27}$ is the equation of a line T.

Find the coordinates of the only point of T which is invariant (i.e. is its own image) under S_K and find the equation of the image of T under S_K.

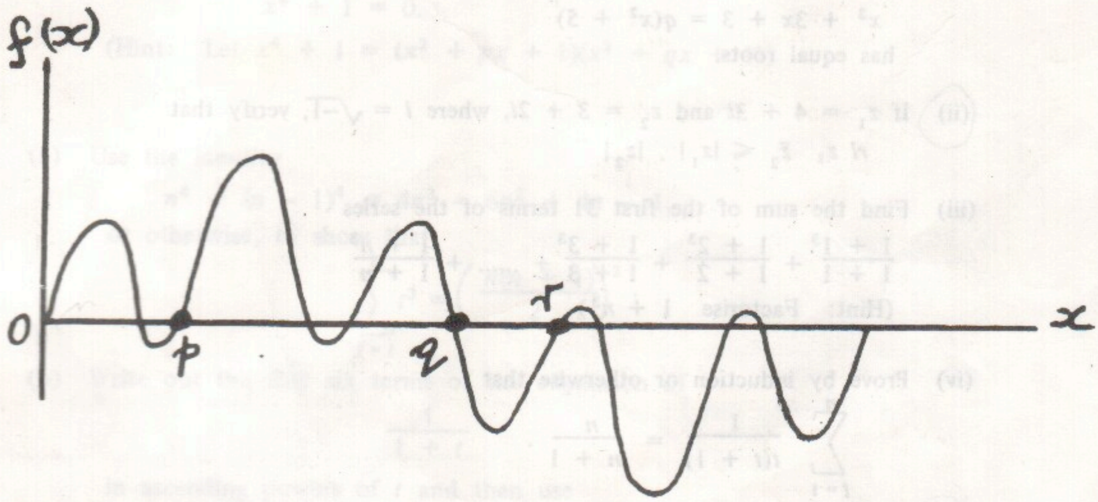
7. (a) Express $\tan 75°$ in the form $p + \sqrt{q}$, where $p, q \in \mathbb{N}$.

(b) Use your tables to find one value of A for which

$$\frac{2}{1 - \tan A} = 3 \tan \frac{A}{2}.$$

(c) Find the period, k, of the function

$$f : x \to \sin 3x \cos 2x.$$



The diagram is a rough graph of the function f in the domain $0 \leqslant x \leqslant k$. Write down the values of x which correspond to the points p, q, r on the x-axis.

8. Let g be the centroid of an equilateral triangle abc and let R_1, R_2, R_3 be the rotations of the plane about g of angles $120°$, $240°$, $360°$, respectively. Assuming that the composition of functions is associative, verify that

$$\{R_1, R_2, R_3\} \text{ under composition}$$

is a group.

If $X \in \{R_1, R_2, R_3\}$, solve $R_2^{152} \circ X = R_1^{230}$

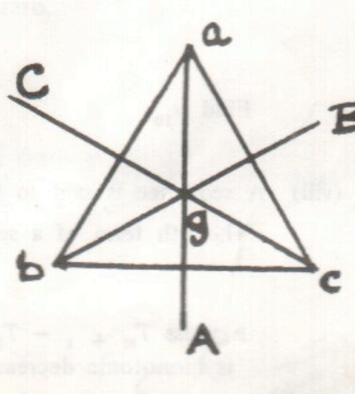
Let S_A, S_B, S_C be the axial symmetries of the plane in A, B, C, respectively. Using the definition of a rotation as the composition of two axial symmetries, or otherwise, prove that

$$\{R_1 \circ S_A, R_2 \circ S_A, R_3 \circ S_A\} = \{S_A, S_B, S_C\}$$

and hence deduce that

$$\{R_1 \circ S_A, R_2 \circ S_A, R_3 \circ S_A\} \text{ under composition}$$

is not a group.



OR

8. Find the coordinates of vertex and focus of the parabola

$$2y^2 - 10y + 18x + 53 = 0.$$

Find also the equation of the directrix and the equation of the axis and draw a rough sketch of the parabola.

A line through the focus perpendicular to the axis cuts the parabola at p and q. Find the equations of the tangents T_1, T_2 to the parabola at these points.

K is a circle whose centre is the vertex of the parabola and whose radius is the distance between the vertex and the focus. Prove that T_1 and T_2 intersect K at the extremities of one of its diameters.