46232

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LEAVING CERTIFICATE EXAMINATION, 1980

MATHEMATICS - HIGHER LEVEL - PAPER I (300 marks)

FRIDAY, 13 JUNE - MORNING, 9.30 to 12.00

Attempt **QUESTION 1** (100 marks) and **FOUR** other questions (50 marks each)

1. (i) Solve the simultaneous equations

$$\frac{1}{x} = \frac{2}{3}$$

$$\frac{1}{x+y} = \frac{2}{5}$$

$$\frac{1}{x+y+z} = 1$$

(ii) Write down the middle term in the expansion of $(3 + \frac{1}{4})^8$. Give your answer in the form $\frac{a}{b}$ where $a, b \in \mathbb{N}$.

(iii) 15 people take part in a chess competition. Each player plays every other player once only. How many games are played ?

(iv) Write the line $x = \dfrac{3-t}{1+t}$, $y = \dfrac{5t}{1+t}$ in the form $ax + by + c = 0$.

(v) Find the equation of the circle which passes through the point of intersection of the circle $x^2 + y^2 - 4x + 6y - 6 = 0$ and the line $2x + y - 5 = 0$ and which contains the origin.

(vi) If $\begin{pmatrix} 2 & 3 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} u \\ v \end{pmatrix}$ and $\begin{pmatrix} 1 & 1 \\ 5 & -2 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 14 \\ 21 \end{pmatrix}$, find the value of x and the value of y.

(vii) Find $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}\begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix}$ and use your result to find $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}^6$.

(viii) Write down a function of the form $a + b\sin cx$ which has period $\frac{2\pi}{3}$ and range $[-1, 7]$.

(ix) Find $\lim\limits_{x \to 0} \dfrac{x \sin 2x}{\cos x - \cos 5x}$.

(x) S is the set of all subsets of $\{1, 2, 3, 4\}$ and Δ is symmetric difference. In the group S, Δ write down the inverse of $\{1, 2\}$ and hence solve for X the equation

$$\{1, 2\} \Delta X = \{1, 2, 3\} .$$

OR

(x) Find the equation of the parabola with focus $(2, 1)$ and directrix $y + 1 = 0$.

OVER →

2. (a) Reduce the three equations

$$2x + 2y + 3z = 0$$
$$3x + ty + 5z = 0$$
$$x + 7y + 3z = 0$$

to two equations in y and z and hence find the value of t such that the three equations, above, have a solution other than $x = 0, y = 0, z = 0$. Find one such solution when $z = -4$.

(b) If α, β are the roots of the equation $x^2 - 2px + q = 0$, show that $\alpha^2 + \beta^2 = 4p^2 - 2q$ and express $\alpha^4 + \beta^4$ in terms of p and q.

3. (a) Write out the first four terms in the expansion of $\frac{1}{1-x}$ in ascending powers of x and give the general term.

If $x = \frac{1}{2}$, find the least number of terms of the expansion that must be added to get a sum greater than 1·99.

(b) Write out the first four terms in the expansion of $\sqrt{x - 2y}$ and use your result to evaluate $\sqrt{98}$ correct to five places of decimals.

4. Show that the equation

$$3x^2 + 10xy + 8y^2 + 16x + 26y + 21 = 0$$

represents two straight lines and find the measure of the smaller angle between them.

Find also the equation of the line which contains the point of intersection of the two lines and which is perpendicular to the line $x - 2y + 3 = 0$.

5. Find the slope of the tangent to the circle

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

at the point (x_1, y_1) on the circle and hence deduce that the equation of the tangent to the circle at that point is

$$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$$

If the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ touches both axes, prove that $g^2 = f^2 = c$ and hence find the equations of the two circles which touch both axes and which contain the point $(a, 2a)$.

Tangents are drawn to both these circles at the point $(a, 2a)$. Investigate if the distances of the origin from these tangents are equal.

6. (a) Let f be the rotation about the origin of angle θ and let g be the axial symmetry in the line $y = x$.

(i) Write down the matrix of g.

(ii) If $\begin{pmatrix} \sqrt{3} \\ -1 \end{pmatrix}$ is the image of $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$ under f, find θ and hence write down the matrix of f.

(iii) If $t = g \circ f$ (i.e. g after f), find the matrix of t and hence find the equation of the image of the line $3y - \sqrt{3}\,x = 0$ under t.

(b) If $M = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}$ and $A = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$, find A^{-1} and evaluate $A^{-1} MA$.

Prove that

$$(A^{-1} MA)^8 = A^{-1} M^8 A$$ and hence express M^8 in the form PQR, where P, Q, R are matrices.

7. (a) Prove that $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\dfrac{x+y}{1-xy}$ under the usual conditions and find the angle θ which is represented by

$$\tan^{-1}\tfrac{1}{3} + \tan^{-1}\tfrac{1}{5} + \tan^{-1}\tfrac{1}{7} + \tan^{-1}\tfrac{1}{8}.$$

(b) Write down the period and range of the function

$$x \to -3 - 3\sin(2x+5).$$

(c) f and g are periodic functions of period p. If $F = f.g$ (when $f.g$ is the product of f and g), prove that F is also periodic. Give an example to show that the period of F is not necessarily p.

8. (a) K is the set of rotations of a square about the point of intersection of its diagonals which maps the square onto itself. Show that K is a group under composition and write out its subgroups.

(b) Show that $\{1, 2, 3, 4\}$ (mod 5) is a group under multiplication and find the least value of $n > 1$ for which $2^n = 1$ (mod 5).

(c) Investigate if the group $\{1, 5, 7, 11\}$ (mod 12) under multiplication is isomorphic to either the group in (a) or (b), above.

OR

8. (a) Find the coordinates of the focus and the equation of the directrix of the parabola

$$(x+2)^2 = 8(1-y).$$

(b) Use the Theorem of Pythagoras to show that

$$x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$$

is the square of the length of the tangent from (x_1, y_1) to the circle

$$x^2 + y^2 + 2gx + 2fy + c = 0.$$

p is a point such that its distance from the line $x + 3 = 0$ is equal to the length of the tangent from it to the circle

$$x^2 + y^2 + 4x - 6y - 34 = 0.$$

Show that the locus of p is a parabola.

(c) Show that $(at^2, 2at)$ are the coordinates of a point on the parabola $y^2 = 4ax$.

The parameters t_1 and t_2 define two points p and q on the parabola $y^2 = 4ax$. If $t_2 = -2t_1$, find the equation of the locus of the point of intersection of the tangents to the parabola at p and q and express this equation in terms of x and y.