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LEAVING CERTIFICATE EXAMINATION, 1972

MATHEMATICS - HIGHER LEVEL - PAPER I

(300 marks)

THURSDAY, 8th JUNE - MORNING, 9.45 to 12.15

Six questions to be answered.
All questions are of equal value.
Mathematics Tables may be obtained from the Superintendent.

1. (a) Show that if $x = t + 2$ and $y = 2t - 1$, the point (x, y) will be in the line containing $(0, -5)$ and $(3, 1)$.

   (b) Find the co-ordinates of the orthocentre of the triangle with vertices at $(9, 1)$ $(5, 7)$, $(3, 2)$.

2. Find the equation of the circle containing the points $(4, 2)$, $(6, 4)$, $(2, 4)$. Compute the distance from the centre of the circle to any chord of length $2\sqrt{3}$. Hence, or otherwise, find the equations of the lines with slope $\frac{3}{4}$ from which equal intercepts of length $2\sqrt{3}$ in each case, are cut off by the circle.

3. Find the co-ordinates of the vertex and focus of the parabola $y^2 - 6y - 4x + 17 = 0$. Find the equation of the tangent to the parabola which has slope $\frac{2}{3}$.

   Find the equation of the normal to the hyperbola $\frac{x^2}{6} - y^2 = \frac{1}{2}$ at the point $(-3, -1)$.

4. (a) Explain the difference between

   (i) the result of the multiplication of a vector by a scalar, and

   (ii) the scalar product of two vectors.

   (b) Say whether each of the following expressions represents a vector or a scalar, and give your reasons:

   (i) $\vec{x}.\vec{y} + \vec{y}.\vec{z} + \vec{z}.\vec{x}$

   (ii) $(\vec{x}.\vec{y})(\vec{z}.\vec{x})\vec{z}$

   (iii) $(\vec{x}.\vec{y})\vec{z} + (\vec{z}.\vec{y})\vec{x}$.

   (c) The position vectors of the vertices of a triangle are $\vec{o}$, $\vec{x}$, $\vec{y}$, respectively. Show that its area, A, is given by the formula

$$4A^2 = |\vec{x}|^2 \, |\vec{y}|^2 - (\vec{x}.\vec{y})^2$$

5. (a) (i) If $u_n = \sin^n\theta + \cos^n\theta$, prove that

$$\frac{u_3 - u_5}{u_1} = \frac{u_5 - u_7}{u_3} .$$

   (ii) Prove that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.

   (b) Give the domain and range (principal values) of each of the functions $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$, and sketch the graph of the function in each case.

6. Let $\vec{c}$ be a fixed non-zero vector in Π_o. Prove that the transformation $\vec{x} \to \vec{x} + \vec{c}$ of Π_o is not linear.

   Let $\vec{e}_1$ and $\vec{e}_2$ be the orthonormal vectors $\begin{pmatrix}1\\0\end{pmatrix}$, $\begin{pmatrix}0\\1\end{pmatrix}$ in R^2. Let t be a linear transformation such that $t(\vec{e}_1) = \begin{pmatrix}2\\1\end{pmatrix}$ and $t(\vec{e}_2) = \begin{pmatrix}1\\2\end{pmatrix}$. In a diagram illustrate the square whose vertices are the position vectors $\begin{pmatrix}0\\0\end{pmatrix}$, $\begin{pmatrix}1\\4\end{pmatrix}$, $\begin{pmatrix}4\\-1\end{pmatrix}$, $\begin{pmatrix}5\\3\end{pmatrix}$, and show clearly its image under the transformation t. Is t an isometry ? Explain your answer.

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7. A rotation r with the origin as centre through an angle θ follows a projection p on the x-axis parallel to the y-axis, where these axes are perpendicular to each other. $\vec{e}_1$ and $\vec{e}_2$ are unit vectors chosen along the x-axis and the y-axis, respectively. If the vector $\frac{5}{\sqrt{2}}(\vec{e}_1 + \vec{e}_2)$ is the image of the vector $5\vec{e}_1 + 3\vec{e}_2$, find

(i) the matrix of the projection p,

(ii) the measure of the angle θ,

(iii) the matrix of the rotation r,

and illustrate the vectors $\vec{x}$ such that $r(p(\vec{x})) = 3(\vec{e}_1 + \vec{e}_2)$.

8. What is meant by saying that "$\cos x$ has a period of 2π" ?
If $x \in R$ what is the range of values of each of these:

$\frac{1}{2}\cos 2x$; $\sin x + \cos x$; $\sin x - \cos x$?

Find the least positive period of

$f(x) = 3\sin\left(k - \frac{\pi x}{2}\right)$, k constant.

9. (a) Illustrate on an Argand Diagram

(i) $\frac{1}{3}(2 + i5)$, (ii) the solution of $Z^2 = i$.

(b) Prove that $|Z_1 + Z_2|^2 + |Z_1 - Z_2|^2 = 2[|Z_1|^2 + |Z_2|^2]$.

(c) Find complex numbers $Z = x + iy$ and $W = u + iv$ such that $Z + iW = 1$ and $iZ + W = 1 + i$.

(d) Show by a diagram the solution set of $|Z - 2| = \frac{1}{2}$.

10. A. (i) Use suitable illustrations to show that translations form a commutative group under addition.

(ii) If $M_2 = \{2x | x \in Z\}$, determine whether M_2, + and M_2, x are groups.
If $H = Z \setminus M_2$, is H, x a group ? Explain your answer.

(iii) The following is the Cayley Table of the group G, * of order four:

*	I	a	b	c
I	I	a	b	c
a	a	I	c	b
b	b	c	I	a
c	c	b	a	I

Is G, * isomorphic to the group of rotations of the square (of measure 0°, 90°, 180°, 270°) ?

Explain your answer.

OR

10. B. (i) What do you mean by saying that two events are <u>dependent</u> ?

(ii) A bag contains 5 black balls and 4 white balls. One ball is drawn at a time and not replaced, and this process is continued until all the balls are drawn. What is the probability that the balls drawn are alternately black and white ?

(iii) A survey in a large city indicated that 40% of the men aged over 35 years pay rates. If 10 men aged over 35 years are chosen at random what is the probability that exactly 6 of them do <u>not</u> pay rates ?