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# LEAVING CERTIFICATE EXAMINATION, 1969

## MATHEMATICS ( HONOURS) – PAPER I – (300 marks)

WEDNESDAY, 11th JUNE – Morning, 9.30 to 12

Six questions to be answered.
All questions carry equal marks.
Mathematical Tables may be had from the Superintendent.
R is the set of real numbers.

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1. A buoy of height 12 feet consists of a cone mounted on a heavy cylindrical base, and floats with the cone uppermost. The cone and cylinder are equal in volume and have the same radius – 3 ft. at the base. What is the volume of the submerged portion of the buoy if the buoy floats so that the apex of the cone is 6 feet above the waterline ?

2. (a) (i) Illustrate the polar of a point $x$ with respect to a circle C. Explain the illustration.
   (ii) Prove that if the polar of a point P passes through another point T then the polar of T passes through P.
   (b) ABC is a triangle no side of which passes through the centre of a given circle. The polars of A, B, C with respect to the circle determine a triangle A′B′C′. Prove that the sides of ABC are the polars of A′, B′, C′ with respect to the circle.

3. (a) A pencil OP, OQ, OR, OS is cut by a transversal in the points P, Q, R, S. Prove that

$$\frac{\sin P\hat{O}Q \,.\, \sin R\hat{O}S}{\sin Q\hat{O}R \,.\, \sin S\hat{O}P} = \frac{PQ \,.\, RS}{QR \,.\, SP}$$

   (b) If PQRS is a harmonic range prove

$$\frac{1}{PQ} + \frac{1}{PS} = \frac{2}{PR}\,.$$

4. (a) What loci are represented by the equations
   (i) $(x + y)^2 - b^2 = 0$, (ii) $y(x - b) = 0$.
   Sketch the locus in each case.
   (b) Find the co-ordinates of the centre, and the radius of the circle whose equation is $x^2 + y^2 - 4x - 3y + 6 = 0$.
   Find the equation of a circle which cuts this circle at right angles.

5. (a) For what value of $k$ does the line $y = 3x + k$ touch the parabola $y^2 = 4(x + 1)$ ?
   (b) Calculate the area of the surface enclosed by the parabola $y^2 = 4x$ and the line $x + y = 8$.

6. If the $x$-axis is the initial line
   (a) give the polar co-ordinates of each of the following points
   $(2, 1)$, $(-3, 0)$, $(-2, 2)$, $(1, -\sqrt{3})$
   (b) find the polar equation of the line through the origin which contains the point whose polar co-ordinate is $(5, \frac{2\pi}{3})$
   (c) Sketch the curves
   (i) $r = 2\cos\theta$, (ii) $r = 2\cos\theta + 3$.

7. Four coins were tossed together and the number $x$ of heads resulting was observed. The operation was performed 160 times and the frequencies that were obtained for the different values of $x$ are shown in the following table:

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| frequency of $x$ | 50 | 70 | 30 | 8 | 2 |

(i) Represent the table by a histogram.
(ii) Calculate the mean of the distribution.

Would you suspect that the coins were biased ? If so, what frequencies would you predict for the different values of $x$ had the coins been unbiased ?

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8. (a) Given $\vec{a} = 3i + 4j$; $\vec{b} = 2j - i$; $\vec{x} = \vec{b} - \vec{a}$

(i) Calculate $\vec{a} \cdot \vec{x}$ and state whether it is a vector or a scalar.

(ii) find the angle between $\vec{a}$ and $\vec{b}$.

(b) Given $\vec{y} = \alpha i + \beta j$ and $\vec{z} = \beta i + \alpha j$.

If $\vec{y} \cdot \vec{z} = -1$ find a set of possible values for $\alpha$ and $\beta$.

[Note: $i$ and $j$ ar orthonormal vectors, i.e. $|i| = |j| = 1$ and $i \cdot j = 0$].

9. (a) Show that $f(x) = \frac{1}{2}\cos x$, $-\infty < x < \infty$, is a periodic function, and find its range.

(b) $p(x) = \frac{1}{2}\cos x$ and $q(x) = \sin x$ are periodic, $0 \leqslant x < \infty$. Prove $r(x) = p(x) + q(x)$ is periodic.

(c) $f(x)$ and $g(x)$ are periodic. Under what conditions is $F(x)$ a periodic function where $F(x) = f(x) + g(x)$ ?

10. (a) Find the general solutions of the equations in $\theta$

(i) $\sin\theta = -\frac{1}{2}$, (ii) $\sin\theta = \sin\alpha$.

(b) (i) Use De Moivre's theorem to express $\cos 3\theta$ as a polynomial in $\cos\theta$.

(ii) Prove that $\cos n\theta$ can be expressed as a polynomial in $\cos\theta$.