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M.43

LEAVING CERTIFICATE EXAMINATION, 1966

MATHEMATICS (HONOURS) - PAPER II (300 marks)

MONDAY, 13th JUNE - MORNING 10 to 12.30

Six questions to be answered.
All questions are of equal value.
Mathematical Tables may be obtained from the Superintendent.

1. (a) Show that the set of natural numbers (1, 2, 3 .....) is not closed under subtraction i.e. if a and b are any two elements of the set, show that $(a - b)$ is not necessarily an element of the set.
   (b) State which of the following sets are closed under (i) subtraction, (ii) division (excluding division by zero):-

   the positive rationals, the integers, the complex numbers.
   (c) If $a = 7 + i$, $b = 3 + 4i$, express $\frac{a}{b}$ in the form $x + iy$ (x,y real).
   (d) In the case of each of the following, find the values of x (if any) for which it is true:

   (i) $2x + 5 = 3x + 7$, x natural,
   (ii) $(2x - 7)(3x + 6) = 0$, x integral,
   (iii) $(x - 2)(x^2 + 2x + 2) = 0$, x complex.

2. (a) If $a + \sqrt{b}$ (a,b rational and $\sqrt{b}$ irrational) is a root of $f(x) = 0$, where $f(x)$ is a polynomial with rational coefficients, show that $a - \sqrt{b}$ is also a root.
   (b) If α, β, γ are the roots of $x^3 - 10x^2 + 35x - 43 = 0$, show that $\alpha - 3, \beta - 3, \gamma - 3$ are the roots of $x^3 - x^2 + 2x - 1 = 0$.

   Hence, or otherwise, find the one real root of the equation $x^3 - 10x^2 + 35x - 43 = 0$, correct to one place of decimals.

3. (a) Evaluate $\lim_{n \to \infty} \frac{1 - 3n + n^2}{3 + n - 3n^2}$.
   (b) If ϵ is a positive number and $N > \frac{1}{\epsilon}$, show that $\frac{1}{n} < \epsilon$ for all $n > N$.

   Deduce that the sequence which has $\frac{1}{n}$ as the nth term converges to zero.
   (c) Prove that the series $\sum_{r=1}^{\infty} \frac{x^r}{r!}$ converges for all real values of x.

4. 100 delegates, men and women, attended an international conference. 40 of the delegates were women, 60 of the delegates were Irish and 24 of the delegates wore glasses. 13 of the women wore glasses, 28 of the women were Irish and 18 of the Irish delegates wore glasses. If 8 of the Irish women wore glasses, find how many of the non-Irish men did not wear glasses.

5. (i) Graph the set of ordered pairs (x,y) that satisfy the following simultaneous inequalities:

   $x + y \leqslant 6$; $x \geqslant 2$; $y \geqslant 1$; $x + 4y \leqslant 12$.

   If (x,y) is an element of this set of ordered pairs, find the maximum value of $x + 2y$.
   (ii) A factory manufactures x boxes of chocolate at 10s. each and y boxes at £1 each per month. At least 200 of the 10s. boxes and 100 of the £1 boxes must be manufactured per month and in that time not more than 600 boxes in all can be manufactured. Write down one inequality for each of the following:-

   x, y, $x + y$.

OR

5. (i) Prove that the operation of intersection of sets is associative i.e. prove $A \cap (B \cap C) = (A \cap B) \cap C$, where A,B,C are any sets.
   (ii) If A,B,C are sets such that $B \subset A$ and $C \subset A$, prove that $A \cap B = A \cap C$ implies $B = C$ and illustrate your answer by a diagram.

   State whether the statement
   "For all non-empty (not null) sets A, B, C

   $A \cap B = A \cap C$ implies $B = C$"

   is true or false. Give a reason.

6. (a) Differentiate from first principles $1 - \cos x$ with respect to x.

(b) Differentiate with respect to x:

(i) $\frac{1 - \cos x}{\sin x}$ (ii) $\sqrt{1 + x^2}$ (iii) xe^x (iv) $\log_e x^3$.

7. (a) A piece of wire of fixed length is bent so as to form a rectangle. Show that the rectangle which has the shortest diagonal is a square.

(b) Show that for $0 < x < \frac{\pi}{2}$, $x - \sin x$ is an increasing function of x and deduce that $x > \sin x$ for all $x > 0$.
Hence, or otherwise, prove that for all $x > 0$, $\cos x > 1 - \frac{x^2}{2!}$

8. (a) Evaluate

(i) $\int_{-1}^{+1} x(x^2 - 1)\,dx$ (ii) $\int_0^{\frac{\pi}{4}} \sin 3\theta \cos\theta \, d\theta$ (iii) $\int_0^{\frac{1}{2}} \frac{x\,dx}{1 - x^2}$.

(b) Show that for $m \geqslant 0$, $n \geqslant 0$

$$\int_0^1 x^m(1 - x)^n dx = \int_0^1 x^n(1 - x)^m dx.$$

Hence, or otherwise, evaluate $\int_0^1 x(1 - x)^{12}\,dx$.

9. (a) Six points are taken on a line AB and four on a line CD, no one of the points being on both AB and CD. With those points as vertices find how many triangles of non-zero area can be formed.

(b) Write down the first three terms and the general term in the expansion of $(1 + x)^n$ (n a natural number) and by differentiation, or otherwise, express

$$\sum_{r=1}^{n} r \; {}^nC_r$$

in terms of n.

(${}^nC_r$ denotes the number of Combinations of n things taken r at a time.)

10. (a) A box contains 25 tickets numbered 1 to 25 inclusive. Two of the tickets are drawn at random from the box. What is the probability that the numbers on the two tickets
(i) are even (ii) are greater than 17 (iii) either even or greater than 17 ?

(b) A factory manufactures electric light bulbs. A sample of three bulbs is chosen at random from a large batch of bulbs. If it is assumed that 10% of all the bulbs are defective and that the binomial distribution applies, find the probability that (i) 0 (ii) 1 (iii) 2 (iv) 3 of the chosen bulbs are defective.

<u>OR</u>

10. Find the coordinates of the maximum and of the minimum points, if any, and the points of inflexion of the curve $y = \frac{4 - x^2}{4 + x^2}$.

Express x^2 in terms of y and deduce that $-1 < y \leqslant 1$, where (x,y) is any point on the curve.

Trace the curve, indicating the form of the curve as x tends to infinity.