M.40

A N   R O I N N   O I D E A C H A I S

LEAVING CERTIFICATE EXAMINATION, 1966

MATHEMATICS (HONOURS) – PAPER I – (300 marks)

WEDNESDAY, 8th JUNE – MORNING, 10 to 12.30

Six questions to be answered.
All questions carry equal marks.
Mathematical Tables may be obtained from the Superintendent.

1. (i) A right circular cone has a vertical height h and the radius of its base is r. If the cone has the same volume as a sphere of radius r, find the ratio of h to r.

(ii) A right circular cone is cut by a plane parallel to the base. The base of the frustum so formed has a radius of 4 inches and the top of the frustum has a radius of 1 inch. If the volume of the frustum is 120 cubic inches, find the size of the vertical angle of the cone, correct to the nearest degree.

2. A transversal cuts the sides AB, BC (produced), and CA of a triangle at D, E, F, respectively. Prove

$$\frac{AD}{DB} \cdot \frac{BE}{EC} \cdot \frac{CF}{FA} = -1.$$

G is a point on BC and B, G, C, E is a harmonic range. Prove that AG, BF, CD are concurrent.

3. (a) P is a fixed point outside a given circle of centre O and the polar of P with respect to the circle cuts OP at Q. Prove that any circle through P and Q cuts the given circle orthogonally.

(b) M is any point on the radical axis of a system of non-intersecting coaxal circles. A circle is drawn with centre M and radius equal to the tangent from M to one of the circles. Prove that every circle of the system cuts the circle of centre M orthogonally.

Given two non-intersecting circles and their radical axis, explain how to construct a circle coaxal with them which shall pass through a given point X, using (a) and (b) or otherwise.

4. The co-ordinates of the vertices of a triangle are (4, 9), (–4, 3), (–2, –3). Find the co-ordinates of the circumcentre (the centre of the circumscribed circle).

Prove that the circumcentre lies outside the triangle.

5. (a) What values must the coefficients b, c, f, g, h, have, when $a = 1$, so that the equation $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$ will represent a circle of centre (1, 2) and radius 5 ?

(b) Prove that the circle $x^2 + y^2 + 2x - 6y - 35 = 0$ touches the straight line $x - 2y - 8 = 0$.

6. (a) Find the polar equation of the circle $x^2 + y^2 = a^2$ with the point $(-a, o)$ as pole.

(b) Write down an equation (in cartesian form) which represents a parabola and find the slope of the tangent at a point (x_1, y_1) on the parabola.

P is any point on a parabola and F is the focus. The line through P parallel to the axis of the parabola cuts the directrix at Q. Show that the tangent at P bisects FQ at right angles.

<u>OR</u>

6. (a) Find the polar co-ordinates of the point whose cartesian co-ordinates are $(1, \sqrt{3})$.

(b) Show that the curve whose equation is $r^2 = a^2\cos 2\theta$ is symmetrical about the initial line $\theta = o$. Show also that (for real r, a) the curve is enclosed by the circle $r = a$ and the straight lines $\theta = \frac{\pi}{4}$, $\theta = -\frac{\pi}{4}$, and draw a graph of the curve.

7. (a) If σ is the standard deviation and $\bar{x}$ the arithmetic mean, show that

$$\sigma^2 = \sigma_a^2 - (\bar{x} - a)^2$$

where σ_a^2 is the average of the squared deviations about any number a.

(b) A survey was made of the number of children in each of 100 houses. The following table gives the frequency distribution:

Number of Children in House	0	1	2	3	4	5	6
Number of Houses	5	16	32	23	12	8	4

Draw a diagram to represent the distribution.

Calculate the arithmetic mean, and calculate the standard deviation correct to two significant figures.

8. (a) Write down the components, along the axes of co-ordinates, of the vector $\vec{AB}$ and the vector $\vec{BA}$, where A is the point (1, 1) and B the point (2, 3). What unit vector has the same direction as $\vec{AB}$?

Find the sum and the scalar (dot) product of $\vec{AB}$ and $\vec{BA}$

(b) The position-vectors of three points P, Q, R (i.e. the vectors $\vec{OP}$, $\vec{OQ}$, $\vec{OR}$, where O is the origin) are $\vec{i} + 3\vec{j}$, $4\vec{i} + 7\vec{j}$, $8\vec{i} + 4\vec{j}$, respectively, where $\vec{i}$, $\vec{j}$ are unit vectors at right angles to each other. Find the unit vectors that have the same directions as $\vec{PQ}$ and $\vec{PR}$, and calculate the size of the angle QPR.

9. Throughout this question the domain of x is the real numbers.

(a) What is the range of values (i) of $\sin x$, (ii) of $\sin 2x$, (iii) of $2\sin x$, (iv) of $\sin^2 x$?

Why is $\sin x$ said to be a *periodic* function ?

Write down three functions of x of the form $b\sin^n ax$ where a, b, n are positive integers, as follows:

(i) a function of period 2π and range $[-3, 3]$ i.e. the range is all numbers from –3 to +3, inclusive,
(ii) a function of period π and range $[-1, 1]$,
(iii) a function of period 2π and range $[0, 2]$.

(b) Find the general solution of the equation

$$\sin x + \sin 2x - \sin 4x = \tfrac{1}{2} - \cos 3x.$$

10. (a) Find a value for x such that

$$\sin^{-1}\tfrac{4}{5} + \sin^{-1}\tfrac{5}{13} = \cos^{-1} x.$$

(b) Prove De Moivre's Theorem where the exponent is a positive integer.

Hence, or otherwise, express $\cos 5\theta$ as a polynomial in $\cos\theta$.