A N   R O I N N   O I D E A C H A I S

LEAVING CERTIFICATE EXAMINATION, 1965

MATHEMATICS - GEOMETRY - HONOURS

MONDAY, 21st JUNE - Morning 10 to 12.30

Not more than <u>seven</u> questions to be answered.

Mathematical Tables may be obtained from the Superintendent

1. Three concurrent straight lines are drawn through the vertices A, B, C of a triangle ABC to meet the opposite sides at D, E, F, respectively. Prove that $\frac{BD}{DC}.\frac{CE}{EA}.\frac{AF}{FB} = +1$. State the converse theorem.
   If the incircle of a triangle PQR touches QR, RP, PQ at X, Y, Z, respectively, prove that PX, RZ, QY are concurrent.

(35 marks)

2. (a) If a circle is described through any pair of inverse points with respect to a given circle prove that it will cut the given circle orthogonally.
   (b) If a pair of inverse points are taken with respect to a circle, prove that the angle which the straight line joining them subtends at any point on the circumference of the circle is bisected internally and externally by the straight lines joining that point to the extremeties of the diameter on which the inverse points lie.

(35 marks)

3. Find the coordinates of the feet of the perpendiculars from the point (5, 0) to the sides of the triangle formed by joining the points (4, 3), (-4, 3) and (0, -5), and prove that the feet of the perpendiculars are collinear.

(35 marks)

4. Find the equation of the circle which passes through the points of intersection of the circles $x^2 + y^2 - 4 = 0$ and $x^2 + y^2 - 2x - 4y + 4 = 0$ and which touches the line $x + 2y = 0$.

(36 marks)

5. (a) P is any point on a parabola and M is the foot of the perpendicular from P to the directrix. If S is the focus, prove that the tangent at P bisects the angle SPM.
   (b) Find the equation of the parabola with focus (1, -1) and directrix $x - y = 0$.

(36 marks)

6. A point P moves in the first quadrant so that the product of the perpendiculars from P to the axes of coordinates is equal to the square on the perpendicular from P to the line $x + y = 3$. Show that the locus of P is a circle which touches the axes of coordinates.
   If P moves under the above conditions in the second quadrant, is the locus of P then a circle ? Give a reason for your answer.

(36 marks)

7. In a triangle ABC, using the usual notation, prove that

$$r = \frac{\Delta}{S} = \frac{a \sin\frac{1}{2}B \sin\frac{1}{2}C}{\cos\frac{1}{2}A},$$

and that

$$\cos A + \cos B + \cos C - 1 = \frac{r}{R}.$$

(36 marks)

8. (i) Show that $\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{8}{17} + \sin^{-1}\frac{36}{85} = \frac{\pi}{2}$;
   (ii) Find the general solution of $2\cos x - \sin x = 1$.

(36 marks)