AN ROINN OIDEACHAIS M.40.

LEAVING CERTIFICATE EXAMINATION, 1965

MATHEMATICS – ALGEBRA – HONOURS

WEDNESDAY, 23rd JUNE – Morning, 10 to 12.30

Not more than seven questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

1. (a) Factorise $x^4 + x^2y^2 + y^4$;

(b) Solve the simultaneous equations

$$x^4 + x^2y^2 + y^4 = 21, \qquad x^2 + xy + y^2 = 3.$$

(35 marks)

2. (a) Denoting the cube roots of unity by 1, w, w^2, show (i) that $1 + w + w^2 = 0$ and (ii) that $(1 + w^2)^4 = w$.

(b) The equation $4x^4 - 4x^3 + 17x^2 - 16x + 4 = 0$ has two roots of the form $\pm ai$, where a is real and i represents $\sqrt{-1}$. Find the value of a and solve the equation fully.

(35 marks)

3. (a) Find the sum of the series $1 + 3x + 5x^2 + 7x^3 + \ldots\ldots\ldots + (2n - 1)x^{n-1}$.

(b) The nth term of a series is $\frac{1}{(7n - 4)(7n + 3)}$; find the sum to n terms of the series.

(35 marks)

4. (a) Show that

$${}^{n-1}C_{r-1} = {}^{n-2}C_{r-2} \cdot \frac{n-1}{r-1},$$ where ${}^{n}C_{r}$ represents the number of combinations of n things taken r at a time.

(b) Find the number of ways in which a selection of four letters can be made from the letters of the word MATAMAITIC.

(c) If x is so small that x^2 and powers of x higher than x^2 may be neglected, write $\frac{(1 - 2x)^{\frac{1}{2}}}{3 - x}$ in the form $l + mx$.

(35 marks)

5. (a) Show fully from first principles that $\frac{d}{dx}(\sin x) = \cos x$.

(b) Differentiate with respect to x

(i) $(1 + 2x^2)^2$, (ii) $x \sin 2x$.

(36 marks)

6. When a taxi-service charges at the rate of 1s. 3d. per mile its taxis travel a total of 600 miles per day. For each penny per mile increase in charge the total number of miles travelled per day decreases by 25. What charge per mile would yield the greatest gross income ?

If operating costs are 10d. per mile, what charge per mile would yield the greatest net income ?

(36 marks)

7. (a) Evaluate

(i) $\int_1^4 (x - \frac{1}{x^2})dx$, (ii) $\int_0^2 \sqrt{4 - x^2}\, dx$.

(b) Prove that

$$\int_0^{\frac{\pi}{2}} \cos^m\theta \sin^n\theta d\theta = \int_0^{\frac{\pi}{2}} \cos^n\theta \sin^m\theta d\theta;$$

$[m \geq o,\ n \geq o]$.

Hence, or otherwise, show that

$$\int_0^{\frac{\pi}{2}} \cos^4\theta d\theta = \int_0^{\frac{\pi}{2}} \sin^4\theta d\theta.$$

(36 marks)

8. Trace the curve $\frac{x}{x^2 + 1}$ paying special attention to maximum and minimum points and to points of inflexion.

(36 marks)