AN ROINN OIDEACHAIS. M.43.

LEAVING CERTIFICATE EXAMINATION, 1964.

MATHEMATICS - GEOMETRY - HONOURS.

FRIDAY, 5th JUNE - Morning 10 to 12.30.

Not more than seven questions may be answered.
Mathematical Tables may be obtained from the Superintendent.

1. (a) Prove that the inverse of a circle with regard to a point on its circumference is a straight line.

(b) Prove that if the point Q lies on the polar of the point P with respect to a given circle, then P lies on the polar of Q.

(35 marks.)

2. Prove that, if in a pencil of four rays a transversal parallel to one ray has equal intercepts made on it by the other three rays, then the pencil is harmonic.

(35 marks.)

3. Find the coordinates of the centre of the circumcircle of a triangle the coordinates of whose vertices are $(-1, 2)$, $(1, 0)$, $(-2, -\frac{1}{2})$.

(35 marks.)

4. Show that the quadrilateral whose vertices are $(1, -1)$, $(0, 1)$, $(1, 3)$, $(4\frac{1}{2}, 2\frac{1}{2})$ is cyclic and find the radius of its circumcircle.

(36 marks.)

5. (a)(i) Show how to construct the radical axis of two non-intersecting circles.

(ii) Prove that the three radical axes of three circles taken in pairs are concurrent.

(b) Prove that the circles $x^2 + y^2 - 2x - 8y + 7 = 0$ and $x^2 + y^2 + x - 9y - 2 = 0$ touch internally and find the equation of their common tangent.

(36 marks.)

6. Show that the equation of the tangent at the point $P(x^1, y^1)$ to the parabola $y^2 = 4ax$ is $yy^1 = 2a(x + x^1)$.
If the tangent at P meets the axis of the parabola at T, show that the tangent at the vertex bisects PT. If the normal at P meets the axis at G and if M is the foot of the perpendicular from P to the axis, show that MG(the subnormal) is constant.

(36 marks.)

7. In a triangle ABC prove that

$$\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \quad .$$

If the radii of the four circles that touch the sides of a triangle are in porportion, prove that the triangle is right-angled.

(36 marks.)

8. (a) Show that $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$.

(b) Find the general solution of each of the following equations:-

(i) $\cos\theta + \sin\theta = 0$.
(ii) $\cos\theta - \sin\theta = 1$.

(36 marks.)