L E A V I N G   C E R T I F I C A T E   E X A M I N A T I O N, 1964.

MATHEMATICS — ALGEBRA — HONOURS

TUESDAY, 9th JUNE — Morning, 10 to 12.30.

Not more than <u>seven</u> questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

1. Solve the following simultaneous equations:—

$$x^2 - yz = 3, \ y^2 - zx = 5, \ z^2 - xy = -1.$$

(35 marks.)

2. (i) Find the quadratic factors of $2x^4 - x^3 + x^2 - 3x - 15$.
Hence find the roots of the equation $2x^4 - x^3 + x^2 - 3x - 15 = 0$.

(ii) Express $\sqrt{8 + 6i}$ in the form $a + ib$ where a and b are real and $i = \sqrt{-1}$.

(35 marks.)

3. (i) The nth term of the series $1, -3, -5, - - - - -$ is of the form $an^2 + bn + c$. Find the sum to n terms of the series.

(ii) Show that $\dfrac{4}{(2n - 1)(2n + 1)(2n + 3)} = \dfrac{1}{(2n - 1)(2n + 1)} - \dfrac{1}{(2n + 1)(2n + 3)}$ and

find the sum to n terms of the series $\dfrac{1}{1.3.5} + \dfrac{1}{3.5.7} + - - - - - -.$

(35 marks.)

4. (i) Find the condition that ^nC_r is greater than $^nC_{r-1}$, where ^nC_r denotes the number of combinations of n things taken r at a time.

(ii) In how many ways can 4 books be selected from 10 books so that one particular book is (a) included, (b) excluded ?

(iii) Use a binomial expansion to find, correct to five significant figures, the value of $(0 \cdot 98)^{-1}$.

(36 marks.)

5. Differentiate with respect to x

(i) $a^2 + x^2$; (ii) $\sqrt{a^2 + x^2}$; (iii) $(a^2 - x^2)\sqrt{a^2 + x^2}$; (iv) $\dfrac{\sin x}{\sin x + \cos x}$.

(36 marks.)

6. Find the value of

(i) $\displaystyle\int_1^4 (1 + 3\sqrt{x} - \dfrac{1}{x^2}) dx$; (ii) $\displaystyle\int_0^{\frac{\pi}{2}} \sin^2 x \cos x dx$; (iii) $\displaystyle\int_0^{\frac{\pi}{2}} \cos^2 x dx$; (iv) $\displaystyle\int_0^1 \dfrac{dx}{1 + x^2}$.

(36 marks.)

7. Trace the curve $x^4 - 8x^3 + 18x^2 - 5$.

(36 marks.)

8. A solid object is circular in horizontal section while a central vertical section of the object is given by the equation $x^2 + y - 10 = 0$. The top of the object is being cut away in thin horizontal slices at the rate of one cubic inch per minute. At what rate is the area of the horizontal section increasing at a distance of 4 inches from the top ?

(36 marks.)