

LEAVING CERTIFICATE EXAMINATION, 1963.

MATHEMATICS - GEOMETRY - HONOURS.

MONDAY, 10th JUNE - Morning 10 to 12.30.

Not more than seven questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

1. A transversal LNM cuts the sides AB and AC of a triangle ABC at L and N respectively and the side BC produced at M; prove that

$$\frac{AL}{LB} \cdot \frac{BM}{MC} \cdot \frac{CN}{NA} = -1.$$

(35 marks.)

2. (a) A straight line AB is divided harmonically at C and D, and O is the mid-point of AB. Prove that $OB^2 = OC \cdot OD$.

- (b) If P is the pole of a straight line QR with regard to a given circle, prove that any straight line through P is cut harmonically by P, the straight line QR and the given circle.

(35 marks.)

3. APB is an arc of a circle. A and B are fixed points while P is a variable point. If the bisector of the angle APB meets the circumference at Q, show that Q is a fixed point. Show, also, that $AP + PB = \frac{AB}{AQ} \cdot PQ$.

(35 marks.)

4. The coordinates of the vertices of a triangle are $(0,0)$, $(4,2)$, $(4,-4)$. Find (i) the coordinates of the point of intersection of the medians, (ii) the coordinates of the orthocentre.

(36 marks.)

5. Find the equations of the two circles which touch the x -axis at the point $(2,0)$ and which also touch the line $3x + 4y = 2$. Find, also, the equation of the circle with centre on the line $3x + 4y = 2$ that cuts these two circles orthogonally.

(36 marks.)

6. What is the equation of the tangent at the point (x_1, y_1) to the parabola $y^2 = 4ax$? Show that the two parabolas $y^2 = 8x$ and $x^2 = y$ cut one another at right angles and at an angle $\tan^{-1} \frac{3}{5}$.

(36 marks.)

7. In a triangle ABC, using the usual notation, prove that

$$(i) \frac{\cot A + \cot B}{c^2} = \frac{\cot B + \cot C}{a^2} = \frac{\cot C + \cot A}{b^2};$$

$$(ii) r_1 r_2 r_3 = rs^2;$$

$$(iii) 4Rrs = abc.$$

(36 marks.)

8. (i) Show that $\tan^{-1} \frac{1}{3} + \frac{1}{2} \cos^{-1} \frac{3}{5} = \frac{\pi}{4}$.

- (ii) Find the general solution of the equation $2\cos^2 x + \sin 2x = 2$.

(36 marks.)