

LEAVING CERTIFICATE EXAMINATION, 1963.

MATHEMATICS - ALGEBRA - HONOURS.

WEDNESDAY, 12th JUNE - Morning, 10 to 12.30.

Not more than seven questions may be answered.
 Mathematical Tables may be obtained from the Superintendent.

1. Solve the following simultaneous equations:

$$(x + y)(x + z) = 30, \quad (y + z)(y + x) = 35, \quad (z + x)(z + y) = 42. \quad (35 \text{ marks.})$$

2. (i) Factorise
- $x^2 + 3xy - 4y^2 - 5x - 10y + 6$
- .

(ii) Find the real roots of the equation $x^4 - 8x^3 + 16x^2 - 64x + 64 = 0$. (35 marks.)

3. (i) Prove the binomial theorem in the case of a positive integral exponent.

(ii) Use a binomial expansion to find, correct to five significant figures, the value of $(30)^{\frac{1}{4}}$. (35 marks.)

4. (i) Show that
- $C_r^n = C_r^{n-2} + 2C_{r-1}^{n-2} + C_{r-2}^{n-2}$
- , where
- C_r^n
- denotes the number of combinations of
- n
- things taken
- r
- at a time.

(ii) In how many ways can a committee of five be chosen from four teachers and six students if at least two teachers are to be on the committee? (35 marks.)

5. Sum to
- n
- terms each of the following series:

(i) $1.3 + 3.5 + 5.7 + \dots$

(ii) $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots$ (36 marks.)

6. Prove from first principles (i) that
- $\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
- , where
- u
- and
- v
- are functions

of x , (ii) that $\frac{d}{dx} (\tan x) = \sec^2 x$.

Differentiate with respect to x (i) $\sqrt{1+x^2}$, (ii) $\frac{x}{\sqrt{1+x^2}}$. (36 marks.)

7. Trace the curve
- $y = \frac{x^2 + 1}{x^2 + x + 1}$
- , paying special attention to maximum and minimum points and to infinite branches.

(36 marks.)

8. Find the value of (i)
- $\int_0^1 (2-x)^3 dx$
- , (ii)
- $\int_0^{\frac{\pi}{4}} \tan^2 \theta d\theta$
- , (iii)
- $\int_0^2 \sqrt{4-x^2} dx$
- .

Interpret geometrically your answer to (iii).

(36 marks.)

9. A thin-walled vessel, 3 inches high, is such that its cross-section at a height of
- x
- inches above the bottom of the vessel is a circle of radius
- $(1+x)$
- inches. How long will the vessel take to fill when water is poured into it at the rate of 10 cubic inches per second? At what rate is the level of the water rising when the depth of the water is 2 inches?

(36 marks.)