AN ROINN OIDEACHAIS

LEAVING CERTIFICATE EXAMINATION, 1962.

MATHEMATICS—Algebra—Honours.

WEDNESDAY, 13th JUNE – Morning, 10 to 12.30.

Not more than seven questions may be answered

Mathematical Tables may be obtained from the Superintendent.

1. (a) Factorise $a^3(b - c) + b^3(c - a) + c^3(a - b)$;

   (b) If i denotes $\sqrt{-1}$, express $(\frac{7}{6} + i\sqrt{2})^{\frac{1}{2}}$ in the form $a + ib$, where a and b are real numbers.

(35 marks.)

2. (a) Show that the sum to n terms of the series $1^3 + 2^3 + 3^3 + \ldots$ is $\frac{1}{4}n^2(n + 1)^2$.

   (b) Find the sum to n terms of the series $0.1.2 + 1.2.3 + 2.3.4 + \ldots$

(35 marks.)

3. Find, correct to two places of decimals, the real root of the equation $2x^3 + 3x^2 + 6x - 6 = 0$.

(35 marks.)

4. (a) Prove that $C^n_{r-1} + C^n_r = C^{n+1}_r$, where C^n_r is the number of combinations of n things taken r at a time.

   (b) If the binomial theorem is true for a given positive integral value of the exponent n, say for $n = p$, show that it is also true for $n = p + 1$.

   (c) Use a binomial expansion to evaluate $(128)^{-\frac{1}{3}}$ correct to four places of decimals.

(36 marks.)

5. (a) Find the limiting value of $\frac{t^2 - 4}{t - 2}$, as t approaches 2.

   (b) Differentiate with respect to x

   (i) $\sqrt{x - 1}$ (ii) $x^2\sqrt{x - 1}$ (iii) $\tan x$.

   Give a geometric interpretation of your answer to (iii).

   (c) If $x = a(\theta - \sin\theta)$ and $y = a(1 - \cos\theta)$, show that $y\left[1 + \left(\frac{dy}{dx}\right)^2\right] = 2a$.

(36 marks.)

6. A window is to be made in the form of a rectangle surmounted by a semicircle. The diameter of the semicircle is to be equal to the width of the rectangle and the perimeter of the window is to be of given length. The rectangular part of the window is to consist of clear glass and the semicircular part of coloured glass. Assuming that unit area of clear glass admits twice as much light as does unit area of coloured glass, find what should be the ratio of the height to the width of the rectangle so that the greatest amount of light would be admitted through the window.

(36 marks.)

7. (a) Evaluate each of the following:

$$\int_1^2 \left(x^2 + \frac{1}{x^2}\right) dx; \qquad \int_0^{\frac{\pi}{2}} \sin 3x \,.\, \cos 2x \, dx; \qquad \int_0^1 \frac{x^2\,dx}{\sqrt{1-x^2}} .$$

   (b) Prove that $\int_0^a f(x)dx = \int_0^a f(a - x)dx$ and deduce that $\int_0^{\frac{\pi}{2}} \sin^n\theta d\theta = \int_0^{\frac{\pi}{2}} \cos^n\theta d\theta$

(36 marks.)

8. Trace the curve $y = \pm(x - 2)^2(x - 1)^{\frac{1}{2}}$, paying special attention to maximum and minimum points and to points of inflexion.

(36 marks.)