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(Department of Education)

LEAVING CERTIFICATE EXAMINATION, 1961.

MATHEMATICS—Geometry—Honours.

FRIDAY, 9*th JUNE*.—Morning, 10 to 12.30.

Not more than *seven* questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

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1. (*a*) A, C, B, D are four points on a straight line. If C and D divide AB harmonically, prove that A and B divide CD harmonically.

   (*b*) If four concurrent lines are such that one transversal is cut by them harmonically, prove that any other transversal will be cut by them harmonically.

[35 marks.]

2. The internal bisector of the angle A of a triangle ABC meets the base BC at D. The circumcircle of the triangle ADB cuts AC at Q and the circumcircle of the triangle ADC cuts AB at P. Prove that BP=CQ.

[35 marks.]

3. The lines $4x-3y+1=0$ and $8x-5y-1=0$ intersect at P. Find the co-ordinates of P.

If O is the origin, find (i) the equation of the line through P perpendicular to OP, (ii) the ratio of the areas of the triangles OPA and OPB, where A and B are the points in which the perpendicular through P to OP meets the x and y axis, respectively.

[35 marks.]

4. Find the co-ordinates of the centre of the circle

$$x^2+y^2-4x-2y+4=0$$

and the equations of the tangents to the circle from the origin.

Find, also, the radius of the circle which is orthogonal to the given circle and with centre at the point $(-1, 2)$.

[36 marks.]

5. (*a*) A is a fixed point within a given circle. Show how to construct the inverse of A with respect to the circle.

(*b*) P is a variable point on a straight line which cuts a given circle and which does not pass through the centre of the circle. Show that the locus of the inverse of P with respect to the given circle is another circle.

If the equation of the given circle is $x^2+y^2=4$, and if the equation of the straight line is $2x-y=1$, find the equation of the inverse circle.

[36 marks.]

6. Find the equation of the parabola with focus $(1, -2)$ and directrix $x+y-2=0$.

Prove that the tangents at the extremities of any focal chord of a parabola intersect at right angles on the directrix.

A variable circle passes through the point $(4, 0)$ and touches the y-axis. Find the equation of the locus of its centre and show that the locus is a parabola. Find also the co-ordinates of the vertex of the parabola.

[36 marks.]

7. (*a*) In a triangle ABC, using the usual notation, prove

(i) that $r_1 = \dfrac{\Delta}{s-a}$,

(ii) that the triangle is right-angled if $r_1 - r = r_2 + r_3$.

(*b*) The median from A, of the triangle ABC, meets BC at D. Prove that $\cot ADC = \frac{1}{2}(\cot B - \cot C)$.

[36 marks.]

8. (*a*) Prove that $\tan^{-1}\dfrac{a}{b} + \tan^{-1}\dfrac{b-a}{b+a} = \dfrac{\pi}{4}$;

(*b*) (i) Show that $\tan\dfrac{3\pi}{8} = \sqrt{2}+1$;

(ii) Find the general solution of the equation $\tan\theta + \sec 2\theta = 1$.

[36 marks.]