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(Department of Education)

LEAVING CERTIFICATE EXAMINATION, 1960.

MATHEMATICS—Geometry—Honours.

FRIDAY, 10*th* *JUNE*.—MORNING, 10 TO 12.30.

Not more than *seven* questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

1. (*a*) If a transversal cuts the sides BC, CA, AB of a triangle ABC at L, M, N, respectively, prove that

$$\frac{BL}{LC} \cdot \frac{CM}{MA} \cdot \frac{AN}{NB} = -1.$$

(*b*) A straight line AB is divided harmonically at C and D. If O is the mid-point of AB, prove that $OB^2 = OC.OD$.

[35 marks.]

2. A circle touches another circle internally at O. A chord AB of the larger circle touches the smaller circle at C and the straight lines OA, OB cut the smaller circle at P and Q, respectively. Prove that

OP : OQ=AC : CB.

[35 marks.]

3. If a circle cuts each of two circles of a system of non-intersecting coaxal circles orthogonally, prove that it cuts all circles of the system orthogonally and that it passes through the limiting points of the system.

Prove that the limiting points of the system are inverse points with regard to any circle of the system.

[35 marks.]

4. The coordinates of the vertices A, B, C of the parallelogram ABCD are (0, 0), (3, 2), (2, −1) respectively. Find the coordinates of the vertex D. Find also (i) the equation of the perpendicular through B to AC, (ii) the length of AC, (iii) the area of the parallelogram.

[36 marks.]

5. Find the equation of the circle which passes through the origin and through the points of intersection of the circles

$x^2+y^2+2x-y-3=0$ and $x^2+y^2+4x-y+3=0$.

Find also the equation of the tangent at the origin to the required circle.

[36 marks.]

6. (*a*) P is any point on the parabola $y^2=4ax$ and F is the focus of the parabola. If the tangent at P cuts the directrix at K, prove that KF is perpendicular to PF.
(*b*) Find the equation of the parabola having its focus at the point (3, 1) and having the line $x-2y+3=0$ as its directrix.
(*c*) The distance of a variable point P from the line $x=5$ is the same as the length of the tangent from P to the circle $(x-13)^2+y^2=144$. Find the locus of P and show that it it is a parabola.

[36 marks.]

7. In a triangle ABC, using the usual notation, prove that
(i) $\cos\frac{A}{2}=\sqrt{\frac{s(s-a)}{bc}}$;
(ii) $\triangle=\frac{1}{2}bc\sin A=\sqrt{s(s-a)(s-b)(s-c)}$;
(iii) if the internal bisector of the angle A meets the base at D,

$$AD=\frac{2bc}{b+c}\cos\frac{A}{2}.$$

[36 marks.]

8. (*a*) Find the general solution of the equation $2\cos\theta+\sin\theta=2$.
(*b*) If $2\cos(x+\theta)\cos(x-\theta)=1$, show that

$$\tan^2 x=\frac{1-\tan^2\theta}{1+3\tan^2\theta}.$$

[36 marks.]