

AN ROINN OIDEACHAIS

(Department of Education)

LEAVING CERTIFICATE EXAMINATION, 1960.

MATHEMATICS—Algebra—Honours.

TUESDAY, 14th JUNE.—MORNING, 10 TO 12.30.

Not more than seven questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

1. When is a function said to be (i) homogeneous, (ii) symmetric ?
Factorise

(a) $(x+y+z)^5 - x^5 - y^5 - z^5$;

(b) $15x^2 - 11xy + 2y^2 + 8x - 2y - 12$.

[35 marks.]

2. (a) Solve the simultaneous equations

$$x^2 + y^2 + z^2 = 133, \quad y + z - x = 7, \quad yz = x^2.$$

- (b) If ai is a root of the equation $6x^4 - 5x^3 + 7x^2 - 5x + 1 = 0$
where a is real and i denotes $\sqrt{-1}$, find a and solve
the equation fully.

[35 marks.]

3. Find the sum to n terms of the series

(i) $2 \times 4 + 3 \times 5 + 4 \times 6 + \dots$

(ii) $\frac{1}{2 \times 4} + \frac{1}{3 \times 5} + \frac{1}{4 \times 6} + \dots$

[35 marks.]

4. (a) In how many ways can the letters of the word *addenda* be
arranged when all are taken at a time ?

In how many of these arrangements

(i) do the three *d*'s come together,

(ii) do the three *d*'s not come together ?

- (b) Use a binomial expansion to evaluate $\sqrt[3]{998}$ correct to five
places of decimals.

[36 marks.]

5. (a) Assuming that the limiting value of $\frac{\sin \Delta x}{\Delta x}$ is unity, as

$\Delta x \rightarrow 0$, prove from first principles that $\frac{d}{dx}(\tan x) = \sec^2 x$.

- (b) Differentiate with respect to x

(i) $x \tan^2 x$,

(ii) $\frac{\sqrt{(2x-a)}}{x}$.

[36 marks.]

6. (a) An open rectangular tank made of thin metal is to have a length l and a capacity v . If its height is x , find, in terms of v and l , for what value of x its surface area is a minimum and show that the breadth of the tank is then twice its height.

- (b) The function $x^2 + px + q$ is zero when $x=1$ and also it is increasing when $x=1$. If the minimum value of the function is -4 , find p and q .

[36 marks.]

7. Evaluate each of the following:

$$\int_1^2 (x+4)^2 dx; \quad \int_0^{\frac{\pi}{4}} \tan^2 x dx;$$

$$\int_0^{\frac{\pi}{2}} \sin^3 \theta \cos^2 \theta d\theta; \quad \int_0^a x^2(a^2-x^2)^{\frac{1}{2}} dx.$$

[36 marks.]

8. Trace the curve $y=(x-1)^2(2x-5)$, referring in particular to the maximum and minimum points and to the point of inflexion.

Find the area enclosed between the curve, the y -axis and the line $y+1=0$.

[36 marks.]