AN ROINN OIDEACHAIS

(Department of Education)

LEAVING CERTIFICATE EXAMINATION, 1959.

MATHEMATICS—Geometry—Honours.

THURSDAY, 4th *JUNE*.—Morning, 10 to 12.30.

Not more than *seven* questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

1. P is a point inside a given circle. Show, with proof, how to find the polar of P with respect to the circle.

Prove that any chord through P is cut harmonically by P, the polar of P and the circumference of the circle.

[35 marks.]

2. Show, with proof, (i) that the inverse of a circle with regard to a point on its circumference is a straight line, and (ii) that the inverse with regard to a point outside the circle is another circle.

Two circles cut orthogonally at P and Q. Taking P as the centre of inversion, show that the circles invert into two perpendicular straight lines.

If the centres of the circles are A and B, respectively, show that the inverses of A, B and Q lie on a straight line.

[35 marks.]

3. From a given point P tangents PS, PT are drawn to a given circle and PQR is any secant through P cutting the circumference of the circle at Q and R. If X is the mid-point of QR, show that $\angle PXS = \angle PXT$. Show, also, that PX is directly proportional to the sum of XS and XT.

[35 marks.]

4. Prove that the angle between the two straight lines $y = m_1x + c_1$ and $y = m_2x + c_2$ is $\tan^{-1}\dfrac{m_1 - m_2}{1 + m_1m_2}$.

The equations of two straight lines are $3x + y + 12 = 0$ and $x + 2y - 1 = 0$. Find (i) the acute angle between them, (ii) the distance of their point of intersection from the origin, (iii) the area of the triangle formed by them and the x-axis, (iv) the equations of the lines drawn perpendicular to them through the point (2, 3).

[36 marks.]

5. Find the equation of the circle which passes through the points (0, 2), (1, 5) and has its centre on the line $x+5y-15=0$.

Show that the point (2, 6) lies on the circle and find the equation of the tangent to the circle at this point. Find, also, the equation of each of the tangents from the origin to the circle.

[36 marks.]

6. The tangent to a parabola at a point P meets the axis of the parabola at T; prove that the tangent at the vertex bisects PT.

Find the coordinates of (i) the vertex, (ii) the focus, of the parabola $(y-4)^2=2(x-3)$. Find, also, the equation of the directrix of the parabola.

[36 marks.]

7. In a triangle ABC, using the usual notation, prove that—

(i) $r_1=\dfrac{\Delta}{s-a}$;

(ii) $a(rr_1+r_2r_3)=b(rr_2+r_3r_1)=c(rr_3+r_1r_2)$;

(iii) $a\cos A+b\cos B+c\cos C=4R\sin A\sin B\sin C$.

8. (*a*) Solve the equation $\sin^{-1}x+\sin^{-1}(1-x)=\cos^{-1}x$.

(*b*) Find the general solution of the equation—
$\sin 2\theta\sec 4\theta+\cos 2\theta=\cos 6\theta$.

[36 marks.]