

# AN ROINN OIDEACHAIS

(Department of Education).

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LEAVING CERTIFICATE EXAMINATION, 1959.

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## MATHEMATICS—Algebra—Honours.

MONDAY, 8th JUNE.—MORNING, 10 TO 12.30.

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Not more than *seven* questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

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1. Solve the simultaneous equations

$$4x + y = xy,$$

$$3z + 2x = 2zx,$$

$$3y + 2z = yz.$$

[35 marks.]

2. If  $w$  and  $w^2$  are the imaginary cube roots of unity, show that  $1 + w + w^2 = 0$ .

Show, also, that  $w$  and  $w^2$  are roots of the equation

$$8x^4 + 6x^3 + 3x^2 - 5x - 3 = 0.$$

Hence, or otherwise, factorise  $8x^4 + 6x^3 + 3x^2 - 5x - 3$ .

[35 marks.]

3. (a) Prove that  ${}^nC_r = {}^nC_{n-r} = \frac{n!}{r!(n-r)!}$

(b) A committee of *six* is to be formed from four men and seven women. In how many ways can this be done if there are to be (i) two men and four women on the committee, (ii) at least two men on the committee.

[35 marks.]

4. (a) Show that the sum of the squares of the first  $n$  natural numbers is  $\frac{1}{6}n(n+1)(2n+1)$ .

Find the sum to  $n$  terms of  $1^2 + 3^2 + 5^2 + 7^2 + \dots$

(b) Use the binomial theorem to find the value of  $\sqrt{30}$  correct to *three* places of decimals.

[36 marks.]

5. (a) Prove from first principles that

$$\frac{d}{dx} \left( \frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}, \text{ where } u \text{ and } v \text{ are functions of } x.$$

(b) Differentiate with respect to  $x$

(i)  $\frac{3x-1}{4x+3}$ ,

(ii)  $\sqrt{(a^2-x^2)}$ ,

(iii)  $\tan^2(5x+7)$ .

(c) If  $y=p \sin(ax+b)$ , show that  $\frac{d^2y}{dx^2} + a^2y = 0$ .

[36 marks.]

6. A vessel in the shape of a right circular cone is placed so that it is standing on its apex with its axis vertical and water is flowing into it at the rate of one cubic foot per minute. The vertical height of the vessel is 2 feet and the diameter of its base is also 2 feet. When the vessel is one-eighth full, find (i) the rate at which the water is rising, (ii) the rate at which the free surface area of the water is increasing.

[36 marks.]

7. (a) Evaluate

(i)  $\int_1^4 \frac{3x-2\sqrt{x}}{x} dx$ ;

(ii)  $\int_0^a x(a^2-x^2)^{\frac{1}{2}} dx$ ;

(iii)  $\int_{\frac{\pi}{4}}^{\frac{\pi}{8}} \cos 3x \cos x dx$ .

(b) If  $y = \sqrt{1-x^2}$ , show that  $\int_0^1 y dx = \frac{\pi}{4}$

and interpret the result geometrically.

[36 marks.]

8. (a) Find the area enclosed between the curves  $y^2=4x$  and  $x^2=4y$ .

(b) By rotating a right-angled triangle about one of the sides containing the right angle, prove, using the usual notation, that the volume of a cone is  $\frac{1}{3}\pi r^2 h$ .

[36 marks.]