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(Department of Education).

LEAVING CERTIFICATE EXAMINATION, 1958.

MATHEMATICS—Geometry—Honours.

THURSDAY, 12th JUNE.—MORNING, 10 TO 12.30.

Not more than *seven* questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

1. Prove that if P is any point on the polar of Q with respect to a given circle, the polar of P with respect to the same circle passes through Q.

ABC is a triangle no side of which passes through the centre of a given circle. The polars of A, B, C with respect to the circle form a triangle $A^1 B^1 C^1$. Prove that the sides of ABC are the polars of A^1, B^1, C^1, with respect to the circle.

[35 marks.]

2. If a transversal meets the sides BC, CA, AB of a triangle ABC at D, E, F, respectively, prove that

$$\frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = -1.$$

The inscribed circle of a triangle ABC touches BC, CA, AB at X, Y, Z, respectively; YZ produced meets CB at T. Prove that C, X, B, T is a harmonic range.

[35 marks.]

3. A straight line AB of unit length is divided into two parts at X so that AB . BX=AX^2. Calculate the length of AX, giving the result in surd form.

An isosceles triangle ABC is then constructed so that AB=AC and BC=AX. Show that the angle BAC is equal to 36° and hence find the value of cos36° in surd form.

[35 marks.]

4. Show that the equation of the straight line through the points $(x_1, y_1), (x_2, y_2)$ is $\dfrac{y-y_1}{y_2-y_1}=\dfrac{x-x_1}{x_2-x}$.

The co-ordinates of the vertices of a triangle are (0, 0), (2, 4), (4, 1); find the co-ordinates of the orthocentre of the triangle.

[36 marks.]

5. Find the radius and the co-ordinates of the centre of the circle $x^2+y^2-2x-4y+4=0$.

Show that the circles

$$x^2+y^2-2x-4y+4=0 \text{ and } x^2+y^2-4x+2y-4=0$$

intersect orthogonally. Find the equation of another circle coaxial with these two circles and passing through the point (0, 5).

[36 marks.]

6. (*a*) The focus of a parabola is (0, 1) and the directrix is the straight line $4x-3y=1$. Find the equation of the parabola.

(*b*) Show that the equation of the tangent to the parabola, $y^2=4ax$, at a point $P(x_1, y_1)$ is $yy_1=2a(x+x_1)$.
If the tangent at P meets the axis of the parabola at T, show that ST=SP, where S is the focus of the parabola.

[36 marks.]

7. In a triangle ABC, using the usual notation, prove :—

(i) $\triangle=\frac{abc}{4R}$;

(ii) $\text{Sin}\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{bc}}$;

(iii) $r=4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$.

[36 marks.]

8. (i) If $A+B+C=90°$, prove that

$$\tan A\tan B+\tan B\tan C+\tan C\tan A=1 ;$$

(ii) Show that $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$;

(iii) Find the general solution of the equation

$$\sin 2\theta+\cos 2\theta=\sin\theta+\cos\theta.$$

[36 marks.]