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(Department of Education).

LEAVING CERTIFICATE EXAMINATION, 1958.

MATHEMATICS—Algebra—Honours.

MONDAY, 16th JUNE.—MORNING, 10 TO 12.30.

Not more than *seven* questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

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1. Show that $x^3+6x^2-45x-189=0$ has a root between 6 and 7, and find this root correct to *two* places of decimals.

[35 marks.]

2. (i) The nth term of a series is 3^n+3n. Find the sum to n terms of the series.

   (ii) Sum to n terms $\frac{1}{3\cdot10}+\frac{1}{10\cdot17}+\frac{1}{17\cdot24}+\ldots$

[35 marks.]

3. (i) If $x=1+2i$ and $y=3+i$, express x^2+y^2 and $\frac{y}{x}$ in the form $a+bi$, where a and b are real numbers and i denotes $\sqrt{-1}$.

   (ii) Find the quadratic factors of $x^4+4x^3+5x^2+2x-2$, and hence find the roots of the equation
   $$x^4+4x^3+5x^2+2x-2=0.$$

[35 marks.]

4. Prove the binomial theorem in the case of a positive integral exponent.

Write down the first four terms and the general term in the expansion of $(1-x)^{\frac{1}{3}}$, and find the value of $(994)^{\frac{1}{3}}$ correct to *six* places of decimals

[36 marks.]

5. (*a*) Show from first principles that $\frac{d}{dx}(\sin 2x)=2\cos 2x$.

(*b*) Differentiate (i) $(x-a)^2$, (ii) $\frac{1+x^2}{(x-a)^2}$, with respect to x.

(*c*) If $x=\sin t$, $y=\sin nt$, show that

$$(1-x^2)\left(\frac{dy}{dx}\right)^2=n^2(1-y^2).$$

[36 marks.]

6. A wire of given length is divided into two pieces, one forming an equilateral triangle and the other forming a circle. If the sum of the areas of the triangle and circle is a minimum, find the ratio of the lengths of the two pieces.

[36 marks.]

7. (*a*) Evaluate :

(i) $\int_1^2\left(x^3-\frac{1}{x^3}\right)dx$; (ii) $\int_0^{\frac{\pi}{2}}\cos^2\theta\sin\theta d\theta$; (iii) $\int_0^1\frac{xdx}{\sqrt{(1+x^2)}}$.

(*b*) Show that $\int_{-a}^{+a}(mx+c)dx$ is independent of m and interpret the result geometrically.

[36 marks.]

8. Sketch the curve $y^2=x^2(1-x^2)$, paying special attention to maximum and minimum points. Find the area of one loop of the curve

[36 marks.]