AN ROINN OIDEACHAIS.

(Department of Education).

LEAVING CERTIFICATE EXAMINATION, 1957.

MATHEMATICS—Geometry—Honours.

THURSDAY, 6th JUNE.—MORNING 10 TO 12.30.

Not more than *seven* questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

1. If the polar of P with respect to a given circle passes through Q, prove that the polar of Q passes through P.

O is a point outside a circle of which AB is a diameter. OA and OB cut the circle again at C, D, respectively, and AD and BC intersect at E. Prove that in the triangle ABE each side is the polar of the opposite vertex with respect to a circle whose centre is O, and which cuts the first circle orthogonally.

[35 marks.]

2. The points P, Q, R, S, of an harmonic range are joined to a point O. A straight line through Q parallel to OS cuts OP at M and cuts OR (produced) at N. Prove that MQ=QN.

Hence, or otherwise, prove that any transversal is cut harmonically by the rays OP, OQ, OR, OS.

ABCD is a quadrilateral. AB and DC produced meet at E while AD and BC produced meet at F. Prove that AC is divided harmonically by BD and EF.

[35 marks.]

3. (i) P is any point on the circumference of a circle of which AB is a chord. The tangents at A and B intersect at C, and D, E, F, respectively, are the feet of the perpendiculars from P to CB, CA, AB, (produced if necessary). Prove that PF is a mean proportional between PD and PE.

(ii) A variable circle touches two unequal fixed circles, centres O and M, externally. When the straight line joining the points of contact is produced prove that it cuts OM, produced, in a fixed point.

[35 marks.]

4. Find the equation of the straight line through the point (1, −3) parallel to $3x+4y=6$, and the equation of the straight line through the origin perpendicular to $x-2y=3$.

Find the equations of the two straight lines through the point (1, 2) each of which makes an angle of 45° with the straight line $x-2y=3$.

[36 marks.]

5. The co-ordinates of A and B are (−1, 4) and (3, 2), respectively. Find the equation of the circle on AB as diameter and prove that it touches the straight line $x+2y=2$.

Find the equations of the tangents to the circle from the point (4, 2) and the equation of the chord of contact.

[36 marks.]

6. (i) P is a point on the parabola $y^2=4ax$ of which $S(a, 0)$ is the focus. The tangent at P cuts the axis of the parabola at T, and the normal at P cuts the axis at G. Prove that S is the middle point of TG.

(ii) Show that $3x-5y+8=0$ is a tangent to the parabola $(x-y)^2+3x-5y+8=0$, and that $x-y=0$ is a straight line parallel to the axis of the parabola.

[36 marks.]

7. (i) In a triangle ABC, using the usual notation prove that $r_1=\frac{\Delta}{s-a}$ and that $r_1r_2+r_2r_3+r_3r_1=s^2$.

(ii) Prove that $\tan^{-1}\frac{1}{13}+\tan^{-1}\frac{7}{11}=2\tan^{-1}\frac{1}{3}$.

(iii) Prove $\sin(A+B)=\sin A\cos B+\cos A\sin B$ in the case where A and B are acute angles.

[36 marks.]

8. (i) In a triangle ABC, prove that

$$\tan A=\frac{a\sin C}{b-a\cos C}$$

(ii) Find the general solution of the equation

$$\sin 5\theta+\cos 5\theta+\sin 3\theta-\cos 3\theta=\sqrt{2}.\sin 4\theta$$

[36 marks.]