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(Department of Education).

LEAVING CERTIFICATE EXAMINATION, 1957.

MATHEMATICS—Algebra—Honours.

TUESDAY, 11*th JUNE*.—Morning, 10 to 12.30.

Not more than *seven* questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

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1. Solve the simultaneous equations

$$\left.\begin{aligned} x+3y-z &= 8 \\ x-y+3z &= 4 \\ xyz &= 6 \end{aligned}\right\}$$

[35 marks.]

2. If w, w^2 are the imaginary cube roots of unity, prove that $1+w+w^2=0$, and that

$$(a+wb+w^2c)+(a+w^2b+wc)=2a-b-c.$$

Factorise fully the expression

$$(a+wb+w^2c)^3+(a+w^2b+wc)^3.$$

[35 marks.]

3. (i) Find the sum to n terms of the series

$$5.7+7.9+9.11+\ldots\ldots$$

(ii) The nth term of a series is $\log_2\left(\dfrac{n+1}{n}\right)$; express the sum of the first fifteen terms in as simple a form as you can.

[35 marks.]

4. Show that $T_{r+1}=x\,.\,\dfrac{n-r+1}{r}\,.\,T_r$, where T_r is the rth term in the binomial expansion of $(1+x)^n$ in ascending powers of x.

Give the first three terms in the binomial expansion of $(1-\frac{2}{5}x)^{-6}$ in ascending powers of x.

Find (i) the greatest coefficient, and (ii) the sum of all the coefficients to infinity, in the expansion of $(1-\frac{2}{5}x)^{-6}$.

[36 marks.]

5. (i) Show that the number of permutations of n things taken r at a time is $\dfrac{\underline{|n}}{\underline{|n-r}}$, and that the number of combinations is $\dfrac{\underline{|n}}{\underline{|r}\ \underline{|n-r}}$

(ii) In how many different orders can four boys and four girls be arranged in line

(a) so that all the boys are together ?

(b) so that two particular boys are not together ?

[36 marks.]

6. (i) Differentiate $x(1-3x)^2$ and $\dfrac{2x}{1-x}$ with respect to x.

(ii) Prove that the limit of $\dfrac{\sin x}{x}$ as x tends to zero is 1, and find the limit of $\dfrac{\sin 3x}{\sin 2x}$ as x tends to zero.

(iii) If $y=\sin x-x\cos x$, show that

$$x\frac{d^2y}{dx^2}-2\frac{dy}{dx}+xy=0.$$

[36 marks.]

7. (i) Find the maximum and minimum points and the point of inflexion on the curve $y=2x^3-3x^2-36x+20$.

(ii) Prove that, for a given volume, a closed cylinder will have the least total surface area when its height is equal to the diameter of its base.

[36 marks.]

8. Evaluate $\int_0^1 x(1-x)dx$ and $\int_0^1 x\sqrt{1-x}\ dx$.

If $y=a\sin x+b\sin 2x$, where a and b are constants, prove that

$\int_0^{\pi} ydx=-\int_{\pi}^{2\pi} ydx$ and $\int_0^{2\pi} y\,.\,\sin x dx=\pi a$.

[36 marks.]