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(Department of Education).

LEAVING CERTIFICATE EXAMINATION, 1956.

MATHEMATICS—Geometry—Honours.

THURSDAY, 7th JUNE.—MORNING, 10 TO 12.30.

Not more than *seven* questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

1. What is meant by the radical axis of two circles ?

Show how to construct the radical axis of two non-intersecting circles, centres A and B , and explain your construction.

If C is a point on AB produced, show, with proof, how to construct a circle with C as centre which shall be coaxal with the two given circles. [35 marks.]

2. (i) Find the inverse of a circle with respect to a point on its circumference.

(ii) If a chord of a circle passes through a fixed point P , prove that the chord is divided harmonically by P and the polar of P .

[35 marks.]

3. On the sides of a triangle ABC equilateral triangles ABP , BCQ , ACR are constructed (externally) ; and the incentres of the equilateral triangles are X , Y , Z respectively. Show that $BY : BC = BX : BP$, deduce that the triangles BXY and BPC are similar, and prove that $XY = XZ = ZY$.

[35 marks.]

4. The co-ordinates of the vertices of a triangle are $(0, 2)$, $(2, 6)$, $(4, -2)$. Find the co-ordinates of the circumcentre.

[36 marks.]

5. Find the equation of the straight line passing through the points $(3, -2)$ and $(2, 2)$, and find the area of the triangle which that line makes with the x and y axes.

Find the equations of the two straight lines through $(3, -2)$ each of which makes a triangle of area 4 units with the x and y axes.

[36 marks.]

6. Find the radius and the co-ordinates of the centre of each of the circles $x^2+y^2-6x-4y+11=0$ and $x^2+y^2+4x+6y-19=0$, and show that the circles touch externally.

Find the equation of the tangent at their point of contact.

[36 marks.]

7. (i) The focus of a parabola is the point $(-1, 2)$ and its directrix is the straight line $x+y+2=0$; find the equation of the parabola.

(ii) $P(x_1, y_1)$ and $Q(x_2, y_2)$ are two points on the parabola $y^2=4ax$ of which $S(a, 0)$ is the focus. If S lies on PQ , show that $y_1y_2=-4a^2$ and that the tangents at P and Q are perpendicular to each other.

[36 marks.]

8. (i) With the usual notation, in a triangle ABC $a=7$, $b=5$, $c=3$; find the angle A and the values of R and r .

(ii) Find the general solution of the equation $\sin 2\theta - \cos \theta = \sin 6\theta - \sin 4\theta - \cos 5\theta$.

[36 marks.]