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(Department of Education).

LEAVING CERTIFICATE EXAMINATION, 1956.

MATHEMATICS—Algebra—Honours.

MONDAY, 11th JUNE.—MORNING, 10 TO 12.30.

Not more than *seven* questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

1. Factorise :

$$ab(a-b)^2 + bc(b-c)^2 + ca(c-a)^2.$$

[35 marks.]

2. (i) Prove that the sum of the squares of the first n natural numbers is $\frac{1}{6}n(n+1)(2n+1)$.

The n th term of a series is $3n^2 - n + 1$; find the sum to n terms.

(ii) Find the least number of terms of the geometric series

$$1 + \frac{1}{2} + \frac{1}{4} + \dots$$

that must be taken so that their sum exceeds 1.99.

[35 marks.]

3. Prove the Binomial Theorem in the case of a positive integral exponent.

Give the first three terms in the binomial expansion of (i) $(1-x)^{\frac{1}{2}}$,
(ii) $(8-12x)^{-\frac{1}{2}}$ in ascending powers of x .

Find the value of $\frac{(3.96)^{\frac{1}{2}}}{(7.88)^{\frac{1}{2}}}$, correct to five places of decimals.

[35 marks.]

4. Find the three roots of the equation $x^3 + 2x^2 - 8x - 13 = 0$, correct to one place of decimals in each case.

[36 marks.]

5. (i) Differentiate $\frac{1}{x^2}$ with respect to x from first principles.

(ii) Differentiate $x \sin x$ and $\frac{x \sin x}{1-x}$ with respect to x .

(iii) What is the limit of $\frac{\cos(x+h) - \cos x}{h}$ as h tends to zero?

[36 marks]

6. (i) Find the maximum and minimum points and the point of inflexion on the curve $y=x^3-3x^2-9x+15$.

(ii) If the curve $y=ax^3+bx^2+cx+d$ (where a, b, c, d are constants) has maximum and minimum points at $x=x_1$ and $x=x_2$ respectively, and a point of inflexion at $x=x_3$, prove that $x_1+x_2=2x_3$.

[36 marks.]

7. Evaluate

$$(i) \int_1^2 (x^2-2x+1)dx;$$

$$(ii) \int_0^1 \frac{x dx}{(x^2+1)^2};$$

$$(iii) \int_0^{\frac{\pi}{2}} \sin 2x \cos x dx.$$

[36 marks.]

8. A vessel in the shape of a right circular cone of vertical angle 60° is standing on a horizontal table with its apex upwards. Water is being poured into the vessel (through a small hole in the apex) at a uniform rate of 6 cubic cms. per sec. Find the rate at which the level of the water in the vessel is rising when the surface of the water is 3 cms. below the apex, and find also the rate at which the area of the surface of the water is decreasing at that stage.

[36 marks.]