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(Department of Education).

LEAVING CERTIFICATE EXAMINATION, 1955.

MATHEMATICS – Geometry—Honours.

FRIDAY, 10th JUNE.—MORNING, 10 TO 12.30.

Not more than *seven* questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

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1. A transversal cuts the sides PQ, QR (produced) and RP of a triangle PQR at the points L, M, N respectively. Prove that

$$\frac{PL}{LQ} \cdot \frac{QM}{MR} \cdot \frac{RN}{NP} = -1$$

[35 marks.]

2. Prove that the rectangle contained by the diagonals of a cyclic quadrilateral is equal to the sum of the two rectangles contained by its opposite sides.

P is any point on the circumcircle of an equilateral triangle ABC. Prove that $PA^2+PB^2+PC^2=2AB^2$.

[35 marks.]

3. A pencil of four rays is cut by a transversal. If the points of section form an harmonic range, prove that the points in which any transversal cuts the rays will form an harmonic range.

[35 marks.]

4. A triangle is bounded by the three straight lines $x+2y=8$, $x-y=2$, $8x+y=-11$. Find the co-ordinates of the orthocentre of the triangle.

Find also the area of the triangle.

[36 marks.]

5. A point P moves in such a way that its distance from the point $(-2, 1)$ is always twice its distance from the point $(1, 4)$. Find the equation of the locus of P and show that the locus is a circle.

Find the radius of the circle and the co-ordinates of the centre.

[36 marks.]

6. Find the equation of the circle which passes through the three points $(4, 1)$, $(-2, 3)$, $(0, -1)$.

Tangents are drawn to the circle from the point $(5, 0)$. Find the equations of the tangents and the co-ordinates of the points of contact.

[36 marks].

7. Prove that the equation of the tangent at a point (x_1, y_1) on the parabola $y^2=4ax$ is $yy_1=2a\,(x+x_1)$.

P is a point on a parabola of which S is the focus. The tangent at P cuts the directrix at R and the axis of the parabola at Q. Prove that the tangent at the vertex bisects PQ and that the angle PSR is a right angle.

[36 marks.]

8. (*a*) In a triangle, using the usual notation, prove that

$$r_1+r_2+r_3-r=4R.$$

(*b*) Prove that $\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{3}{4}-\tan^{-1}\frac{2}{11}=\frac{\pi}{4}$.

(*c*) Find the general solution of the equation

$$\cos\theta+\cos3\theta+\cos7\theta=0.$$

[36 marks.]