AN ROINN OIDEACHAIS

(Department of Education).

LEAVING CERTIFICATE EXAMINATION, 1954.

MATHEMATICS—Geometry—Honours.

FRIDAY, 11*th* *JUNE*.—Morning, 10 to 12.30.

Not more than *seven* questions may be answered.

Mathematical Tables may be obtained from the Superintendent

1. D,E,F are points on the sides AB, BC, CA, respectively, of a triangle ABC such that CD, AE, BF are concurrent. Prove that

$$\frac{AD}{DB} \cdot \frac{BE}{EC} \cdot \frac{CF}{FA}=1.$$

If DE produced cuts AC produced at G, prove that A, F, C, G is an harmonic range.

[35 marks.]

2. A is a point within a given circle: show, with proof, how to construct its polar.

A and B are two points inside a circle centre O. The polar of A with respect to the circle cuts OB produced at L, and the polar of B cuts OA produced at M. Prove that ML is parallel to AB and that the orthocentre of the triangle AOB is the pole of ML.

[35 marks.]

3. (i) A triangle ABC is inscribed in a circle and P is a point on the arc AB remote from C. If R and S are points on the sides BC, AB respectively and T a point on the side CA produced, such that $\angle PRB=\angle PSB=\angle PTA$, prove that R, S, T are collinear.

(ii) Four straight lines (e.g. the sides of a triangle and a transversal) intersect so as to form four triangles. Prove that the circumcircles of the four triangles intersect in a common point.

[35 marks.]

4. Find the equation of each of the following:

(i) the straight line passing through the points (1, 6) and (−3, −2);

(ii) the straight line parallel to $x-4y+3=0$ and cutting the x-axis at $x=5$;

(iii) the straight line perpendicular to $2x-3y+4=0$ and passing through the point (3, −4).

[36 marks.]

5. A circle is represented by the equation

$$(x+1)(x+3)+(y-4)(y+2)=0\,;$$

find the radius and the co-ordinates of the centre, and show that the circle touches the straight line $3x-y+17=0$.

The circle is divided into two segments by the straight line $y=2x$. Show that one of the segments contains an angle of 45°.

[36 marks.]

6. P is the centre of a variable circle which touches the straight line $x+2=0$ and which also touches the circle $x^2+y^2-2x=0$. Find the equation of the locus of P, and show that the locus is a parabola.

Find (i) the equation of the axis of the parabola, (ii) the co-ordinates of the vertex, (iii) the equation of the directrix.

[36 marks.]

7. In a triangle ABC the bisector of the angle CAB meets BC in D, and the bisector of the angle ABC meets CA in E. Prove that

$$\mathrm{AD}=\frac{2bc}{b+c}\cos\frac{\mathrm{A}}{2},$$

and write down the corresponding expression for BE.

If AC>BC, show that $\cos\frac{\mathrm{A}}{2}>\cos\frac{\mathrm{B}}{2}$ and that AD>BE.

[36 marks.]

8. (*a*) In a triangle ABC, using the usual notation, prove that

(i) $r_1r_2+rr_3=ab\,;$

(ii) $a\cos\mathrm{A}+b\cos\mathrm{B}+c\cos\mathrm{C}=4\mathrm{R}\sin\mathrm{A}\sin\mathrm{B}\sin\mathrm{C}$.

(*b*) Find the general solution of the equation

$$\sin\theta=\sin 5\theta-\sin 3\theta.$$

[36 marks.]