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(Department of Education).

LEAVING CERTIFICATE EXAMINATION, 1954.

MATHEMATICS—ALGEBRA—Honours.

TUESDAY, 15*th June*.—MORNING, 10 TO 12.30.

Not more than *seven* questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

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1. Solve the simultaneous equations :—

$$\left.\begin{aligned} x+2y+z &= 3 \\ 2x+3y+4z &= 2 \\ x^2+y^2+3z^2 &= 14 \end{aligned}\right\}$$

[35 marks.]

2. Prove the Binomial Theorem in the case of a positive integral exponent.

Write down the first three terms in the binomial expansion, in ascending powers of x, of each of the following : (i) $(1+2x)^7$; (ii) $(27-3x)^{\frac{1}{3}}$.

Use a binomial expansion go find the value of $\sqrt[3]{3}$ correct to three decimal places.

[35 marks.]

3. Find the sum of each of the following series :—

(i) $1.2+3.5+5.8+\ \ldots\ +(2n-1)(3n-1)$;

(ii) $\frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\ \ldots\ +\frac{n}{2^n}$.

[35 marks.]

4. Find the real root of the equation

$$x^3+2x^2-x-6=0$$

correct to two places of decimals.

Find also the real root of each of the following equations, correct to one place of decimals in each case :—

(i) $(x+1)^3+2(x+1)^2-(x+1)-6=0$;

(ii) $x^3+20x^2-100x-6000=0$.

[36 marks.]

5. (i) Find from first principles the differential coefficient, with respect to x, of: (*a*) $\frac{1}{x}$; (*b*) $\sin^2 x$.

Mention the geometrical significance of your result in (*a*).

(ii) If $y = Ax^n + Bx^{-n}$, show that

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n^2y.$$

[36 marks.]

6. In a circle of unit radius, AB is a diameter fixed in position, CD is a variable chord parallel to AB, and P, Q are the feet of the perpendiculars to AB from C and D respectively. Find

(i) the greatest value of the area of the trapezium ABDC,

(ii) the greatest value of the volume generated by rotating the rectangle PQDC about the line PQ.

[36 marks.]

7. Show, by means of a diagram, how the slope of a curve changes near a point of inflexion, and hence explain how you can find the co-ordinates of such a point from the equation of the curve.

Trace the curve $y = x^3 - x^2 - 5x + 4$.

[36 marks.]

8. Show by integration that the area of a circle of radius a is πa^2.

Evaluate :—

(i) $\int_0^1 x^2(2x-1)dx$;

(ii) $\int_0^{\frac{\pi}{4}} \sin 3x \sin x \, dx$.

[36 marks.]