AN ROINN OIDEACHAIS

(Department of Education).

LEAVING CERTIFICATE EXAMINATION, 1953.

MATHEMATICS—Geometry—Honours.

THURSDAY, 11*th JUNE*.—Morning, 10 to 12.30.

Seven questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

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1. When is a given straight line said to be divided harmonically ?

If a straight line AB is divided harmonically at P and Q, show that the straight line PQ is divided harmonically at A and B.

If A, P, B, Q is a harmonic range, and if O is the middle point of AB, prove that QA.QB=QP.QO.

[35 marks.]

2. Show how to construct the radical axis of two non-intersecting circles. Give proof.

P is a point on the radical axis of a system of non-intersecting coaxal circles, and PT is a tangent from P to one of the circles. With P as centre and PT as radius, a circle is drawn which cuts the line of centres in R and S. Prove that this circle cuts each of the circles of the system orthogonally, and that R and S are fixed points.

[35 marks.]

3. In a triangle ABC, the internal bisector of the angle A meets the base BC in D, and the external bisector of the angle A meets BC produced in K. Prove that $DK=\frac{2abc}{b^2-c^2}$

If AD produced meets the circumference of the circumcircle in E, prove that

$$\frac{AE}{DE}=\frac{(b+c)^2}{a^2}.$$

[35 marks.]

4. If the straight lines $Ax+By+C=0$ and $A_1x+B_1y+C_1=0$ are perpendicular to one another, show that $AA_1+BB_1=0$.

Find the co-ordinates of the orthocentre of the triangle formed by the three straight lines $x+2y+1=0$, $2x+y-1=0$, $x-y+1=0$.

[30 marks.]

5. Find the equation of the circle which passes through the point (1, 0) and which touches the line $3x+2y-4=0$ at the point (2, —1).

Tangents are drawn to that circle from the origin, and also from the point (1, 3). Show that the chords of contact are parallel to one another, and find the distance between them.

[36 marks.]

6. Show that the line $m^2x-my+a=0$ touches the parabola $y^2=4ax$, for all values of m.

Hence, or otherwise, show that two tangents can be drawn to the parabola $y^2=4ax$ from a point outside the parabola.

Find the equation of each of the tangents from the point (3, 4) to the parabola $y^2=4x$.

[36 marks.]

7. In a triangle ABC, using the usual notation, prove

(i) $r_1=\dfrac{\triangle}{s-a}$;

(ii) $R=\dfrac{abc}{4\triangle}$;

(iii) $b^2\sin 2C+c^2\sin 2B=4\triangle$.

[36 marks.]

8. (i) Prove that $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$;

(ii) Find the value of x from the equation

$$\sin^{-1}\frac{2a}{1+a^2}+\tan^{-1}\frac{2x}{1-x^2}=\cos^{-1}\frac{1-b^2}{1+b^2};$$

(iii) Find the general solution of the equation

$$3-2\cos\theta-4\sin\theta-\cos 2\theta+\sin 2\theta=0.$$

[36 marks.]