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(Department of Education).

LEAVING CERTIFICATE EXAMINATION, 1952

MATHEMATICS—Algebra—Honours.

TUESDAY, 17th JUNE.—MORNING, 10 TO 12.30.

Not more than *six* questions may be answered.

Mathematical Tables may be obtained from the Superintendent

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1. If $x^2+y^2+z^2=9$ and $xy+yz+zx=-4$, find the value of $x+y+z$.
Solve the simultaneous equations

$$x^2+y^2+z^2=9,$$
$$xy+yz+zx=-4,$$
$$2x-y+3z=1.$$

[40 marks.]

2. Prove that the sum of the squares of the first n natural numbers is $\frac{1}{6}n(n+1)(2n+1)$.

Find the sum of the first n terms of the series :

(i) $1^2+3^2+5^2+\ldots\ldots\ldots$

(ii) $2.3+3.4+4.5+\ldots\ldots\ldots$

[40 marks.]

3. Use the binomial expansion of $\left(1-\frac{1}{50}\right)^{-\frac{1}{2}}$ to find the value of $\sqrt{2}$ correct to four places of decimals.

Use the Binomial Theorem to find the value of $\sqrt{10}$ correct to four places of decimals.

[40 marks.]

4. How many groups of 5 books can be made from 6 Irish books and 8 English books

(i) if each group is to contain one and only one Irish book,

(ii) if each group is to contain at least one Irish book,

(iii) if not more than one of a particular pair of books may be selected for the same group ?

[42 marks.]

5. If α, β, γ, are the roots of the equation $x^3-3x-4=0$, find the two corresponding equations whose roots are (i) $\alpha-2$, $\beta-2$, $\gamma-2$, and (ii) 10α, 10β, 10γ, respectively.

Show that the equation $x^3-3x-4=0$ has a root lying between 2·1 and 2·2, and find the value of that root, correct to three places of decimals.

[42 marks.]

6. Differentiate from first principles (i) x^3, (ii) $\cos 2x$, with respect to x.

Differentiate with respect to x:

(i) $x\tan 3x$, (ii) $x\sin 2x\tan 3x$.

[42 marks.]

7. Find the points on the curve $y^2=(x-1)(x-2)^2$ at which the tangent is parallel to the x—axis, and sketch the curve.

Find the volume generated by rotating the loop about the x—axis.

[42 marks.]

OR

7. Find the altitude of the cylinder of maximum volume that can be inscribed in a given right circular cone of height h.

[42 marks.]

8. Evaluate:

(i) $\int_1^2 x(x+1)^2 dx$;

(ii) $\int_0^{\frac{\pi}{2}} \sin^2 x\, dx$;

(iii) $\int_0^1 \frac{2x}{\sqrt{1+x^2}}\, dx$;

(iv) $\int_0^{\frac{a}{2}} \frac{dx}{\sqrt{a^2-x^2}}$.

[42 marks.]