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(Department of Education).

LEAVING CERTIFICATE EXAMINATION, 1951.

MATHEMATICS—Geometry—Honours.

WEDNESDAY, 6*th* *JUNE*.—MORNING, 10 TO 12.30.

Six questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

1. If the points of section of a pencil of four rays made by a transversal form a harmonic range, prove that the points of section by any other transversal form a harmonic range.

[40 marks.]

2. Prove that the inverse of a straight line with respect to a point not on the line, is a circle through the centre of inversion. Prove, also, that the line is the radical axis of its inverse and the circle of inversion.

[40 marks.]

3. Show that the angle between the lines $y=m_1x+c_1$ and

$$y=m_2x+c_2 \text{ is } \tan^{-1}\frac{m_1-m_2}{1+m_1m_2}.$$

Show that the equation $x^2+xy-6y^2+7x+31y-18=0$ represents two straight lines, and find the angle between them.

[42 marks.]

4. A and B are two fixed points whose co-ordinates are (0,0), (1,3), respectively, and a point P moves so that $PA : PB = \sqrt{2} : 1$. Find the equation of the locus of P.

Show that the locus is a circle and find the co-ordinates of its centre and its radius. Find, also, the equation of the tangents to the circle from the origin.

[42 marks.]

Or

4. Find the equation of the circle described on the straight line joining the points (1,1), (5, 4) as diameter.

Show that the x-axis is a tangent to the circle, and find the equation of the other tangent from the origin.

[42 marks.]

5. Define a parabola.

The focus of a parabola is situated at the point (2, 3) and $y+3=0$ is the equation of the directrix. Find the equation of the parabola.

Find the focus and directrix of the parabola $2x^2+6y-10x+17=0$.

[42 marks.]

6. (*a*) Using the usual notation, prove that in a triangle ABC

$$r=4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}.$$

(*b*) If D is the middle point of the side BC of a triangle ABC, and if $\triangle$ represents the area of the triangle, prove that

$$\cot ADB=\frac{AC^2-AB^2}{4\triangle}.$$

[42 marks.]

7. (i) Prove that $\sin^{-1}(\sqrt{2}\sin\theta)+\sin^{-1}\sqrt{\cos 2\theta}=\frac{\pi}{2}$;

(ii) Solve $\cos^{-1}\frac{1}{\sqrt{1+x^2}}-\cos^{-1}\frac{x}{\sqrt{1+x^2}}=\sin^{-1}\frac{1+x}{1+x^2}$.

[42 marks.]

Or

7. Find the general solution of each of the equations :

(i) $10\cos x-\sin 2x=12\cos^3 x$;

(ii) $3\sin x-4\cos x=5$.

[42 marks.]